ASTRON. 285
AUG 3 1922

VOLUME LVI NUMBER 1

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

JULY 1922

THE VOLTAGE-CURRENT RELATION IN CENTRAL ANODE PHOTO-ELECTRIC CELLS *Herbert E. Ives and Thornton C. Fry* 1

THE WIDTH OF CERTAIN LINES OF THE SPECTRUM OF HELIUM AS A FUNCTION OF PRESSURE IN THE SOURCE *Lloyd W. Taylor* 16

ON EINSTEIN'S ABERRATION EXPERIMENT *C. V. Raman* 29

THE QUANTUM THEORY OF THE SPECTRUM OF HYDROGEN *H. A. Wilson* 34

INTERFEROMETER OBSERVATIONS OF DOUBLE STARS *Paul W. Merrill* 40

A DETERMINATION OF THE WAVE-LENGTHS OF 118 TITANIUM LINES BY MEANS OF THE INTERFEROMETER *Frederick L. Brown* 53

MINOR CONTRIBUTIONS AND NOTES:
Majorana's Theory of Gravitation, A. S. EDDINGTON, 71

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE MISSION BOOK COMPANY, Shanghai

July 1922 Vol. LVI, No. 1

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

WITH THE COLLABORATION OF

JOSEPH S. AMES, Johns Hopkins University
ARISTARCH BELOPOLSKY, Observatoire de Poulkova
WILLIAM W. CAMPBELL, Lick Observatory
HENRY CREW, Northwestern University
CHARLES FABRY, Université de Paris
ALFRED FOWLER, Imperial College, London
CHARLES S. HASTINGS, Yale University
HEINRICH KAYSER, Universität Bonn
ALBERT A. MICHELSON, University of Chicago
HUGH F. NEWALL, Cambridge University
ERNEST F. NICHOLS, Nela Laboratory, Cleveland
ALFRED PEROT, Paris
CARL RUNGE, Universität Göttingen
HENRY N. RUSSELL, Princeton University
SIR ARTHUR SCHUSTER, Twyford
FRANK SCHLESINGER, Yale Observatory

The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶The subscription price is $6.00 a year; the price of single copies is 75 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Dominican Republic, El Salvador, Bolivia, Colombia, Honduras, Nicaragua, Peru, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Shanghai. ¶Postage is charged extra as follows: for Canada, 30 cents on annual subscriptions (total $6.30); on single copies, 3 cents (total 78 cents); for all other countries in the Postal Union, 50 cents on annual subscriptions (total $6.50), on single copies 5 cents (total 80 cents). ¶Patrons are requested to make all remittances payable to The University of Chicago Press in postal or express money orders or bank drafts.

The following are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 16*s*. each; single copies, including postage, 4*s*. 6*d*. each.

For Japan and Korea: The Maruzen-Kabushiki-Kaisha, 11 to 16 Nihonbashi Tori Sanchome, Tokyo, Japan. Yearly subscriptions, including postage, Yen 13.65 each; single copies, including postage, Yen 1.68 each.

For China: The Mission Book Company, 13 North Szechuen Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, if mailed direct outside of Shanghai, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Ryerson Laboratory, University of Chicago, Chicago, Illinois.

The cable address is "University, Chicago."

The articles in this Journal are indexed in the *Readers' Guide Supplement*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 13, 1918.

PRINTED IN THE U.S.A.

Gen.
Water

CONTENTS

NUMBER I

PAGE

THE VOLTAGE-CURRENT RELATION IN CENTRAL ANODE PHOTO-ELECTRIC CELLS. Herbert E. Ives and Thornton C. Fry 1

THE WIDTH OF CERTAIN LINES OF THE SPECTRUM OF HELIUM AS A FUNCTION OF PRESSURE IN THE SOURCE. Lloyd W. Taylor . . . 16

ON EINSTEIN'S ABERRATION EXPERIMENT. C. V. Raman 29

THE QUANTUM THEORY OF THE SPECTRUM OF HYDROGEN. H. A. Wilson 34

INTERFEROMETER OBSERVATIONS OF DOUBLE STARS. Paul W. Merrill . 40

A DETERMINATION OF THE WAVE-LENGTHS OF 118 TITANIUM LINES BY MEANS OF THE INTERFEROMETER. Frederick L. Brown . . . 53

MINOR CONTRIBUTIONS AND NOTES: MAJORANA'S THEORY OF GRAVITATION. A. S. Eddington 71

NUMBER II

REVISION OF THE SERIES IN THE SPECTRUM OF STRONTIUM. F. A. Saunders 73

FURTHER EVIDENCES ON THE BRIGHTNESS OF THE STARS OF THE NORTH POLAR SEQUENCE. Frederick H. Seares and Milton L. Humason . 84

REVISED MAGNITUDES FOR STARS NEAR THE NORTH POLE. Frederick H. Seares 97

ON THE SPECTRUM OF NEUTRAL HELIUM. Ludwik Silberstein . . . 119

THE VARIABLE DOUBLE STAR X OPHIUCHI. C. H. Gingrich . . . 132

PARALLAXES OF STARS IN THE REGION OF B.D.+31°643. C. H. Gingrich 139

NUMBER III

J. C. KAPTEYN, 1851–1922. A. van Maanen 145

VISIBLE AND INFRA-RED RADIATION OF HYDROGEN. Frederick Sumner Brackett 154

A GENERAL STUDY OF DIFFUSE GALACTIC NEBULAE. Edwin Hubble . 162

INVESTIGATIONS ON PROPER MOTION, SEVENTH PAPER. Adriaan van Maanen. 200

INVESTIGATIONS ON PROPER MOTION, EIGHTH PAPER. Adriaan van Maanen. 208

CORRIGENDA 216

NUMBER IV

PAGE

THE CEPHEID VARIABLE η AQUILAE. Charles Clayton Wylie . . . 217
THE ECLIPSING BINARY σ AQUILAE. Charles Clayton Wylie . . . 232
A SPECTROSCOPIC METHOD OF DETERMINING THE ABSOLUTE MAGNITUDES OF A-TYPE STARS AND THE PARALLAXES OF 544 STARS. Walter S. Adams and Alfred H. Joy 242
THE DISTRIBUTION OF THE VELOCITIES OF STARS OF SPECTRAL TYPES F TO M. Gustaf Strömberg 265
THE APPLICATION OF VACUUM THERMOCOUPLES TO PROBLEMS IN ASTROPHYSICS. Edison Pettit and Seth B. Nicholson 295
THE ELECTRIC FURNACE SPECTRUM OF IRON IN THE ULTRA-VIOLET, WITH SUPPLEMENTARY DATA FOR THE BLUE AND VIOLET. Arthur S. King 318
CHANGES IN THE SPECTROGRAPHIC ELEMENTS OF Y SAGITTARII. John C. Duncan 340
PREPARATION OF ABSTRACTS 344

NUMBER V

ASTRONOMICAL PHOTOGRAPHIC PHOTOMETRY AND THE PURKINJE EFFECT. II. F. E. Ross 345
THE SPARK SPECTRUM OF GALLIUM IN AIR AND IN HYDROGEN. Elias Klein 373
THE ABSENCE OF OXYGEN AND WATER-VAPOR LINES IN THE SPECTRUM OF VENUS. Charles E. St. John and Seth B. Nicholson . . . 380
THE SOURCE OF LUMINOSITY IN GALACTIC NEBULAE. Edwin Hubble . 400
ORBIT OF THE ECLIPSING BINARY TW ANDROMEDAE. Martha Betz Shapley 439
ORBITS OF THE SPECTROSCOPIC BINARIES LALANDE 13792 AND AOE 12584. R. F. Sanford 446
STELLAR SPECTRA OF CLASS S. Paul W. Merrill 457
THE BEHAVIOR OF SPECTRAL LINES AT THE POSITIVE POLE OF THE METALLIC ARC. Paul W. Merrill 475
GEOMETRICAL PROOF FOR THE WADSWORTH CONSTANT DEVIATION SYSTEM. R. C. Gibbs and J. R. Collins 483
REVIEWS: *The Telescope*, Louis Bell (A. S. Y.), 486; *Geschichte und Literatur der veränderlichen Sterne*, G. Müller and E. Hartwig (J. A. Parkhurst), 488.
INDEX 489

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME LVI JULY 1922 NUMBER 1

THE VOLTAGE-CURRENT RELATION IN CENTRAL ANODE PHOTO-ELECTRIC CELLS

By HERBERT E. IVES and THORNTON C. FRY

ABSTRACT

Current-voltage relation in a central-anode vacuum photo-electric cell as a function of the frequency of the exciting radiation.—In the case of some potassium cells it was observed that the higher the frequency of the light the higher the voltage required for saturation. The *mathematical theory* was then developed for the case of a spherical anode of radius b at the center of a spherical cathode of radius a. Electrons emitted from a photo-sensitive spot on the cathode describe elliptic orbits determined in each case by the initial direction and velocity and by the electric field. If the maximum velocity of emission of the electrons for radiation of frequency ν is v_m, where $\frac{1}{2}mv_m^2 = h(\nu - \nu_0)$, the minimum difference of potential for saturation comes out $V_s = h(\nu - \nu_0)(a^2 - b^2)/eb^2$. Assuming that the distribution as to direction follows the cosine law and that the distribution as to energy follows a parabolic law with a maximum at half the limit set by the quantum relation $\frac{1}{2}mv_m^2 = h(\nu - \nu_0)$, then the fraction of saturation for any voltage below V_s is $1 - \frac{1}{4}(a^2 - b^2)/a^2 \{1 - V/V_s\}^2$. For an assumed case of $a/b = 15$ and $\nu_0 = 5 \times 10^{14}$, (6000 A), V_s comes out 40 and 160 volts for radiation of wave-length 5500 A and 4500 A respectively. These differences agree as to order of magnitude with some *experimental observations* shown in Figure 7. This effect should evidently be taken into consideration whenever such cells are used to compare the intensities of radiations of different spectral distributions.

1. *Introduction.*—The type of photo-electric cell most commonly used at the present time consists of a cathode in the form of a spherical shell of sensitive material (e.g., potassium on the inside of a glass bulb) together with a centrally placed anode in the form of a loop or grid. This construction is the exact opposite of that which would be chosen if we desired to reach the saturation current at low voltages. Probably this is not a serious disadvantage in the

majority of uses to which photo-electric cells are put, particularly as most of the cells employed for light-measuring purposes are of the gas-filled type operating on high voltages in order to get the advantage of ionization by collision. What may be a serious matter, however, is the fact that the rate at which saturation is approached depends upon the wave-length of the incident light.

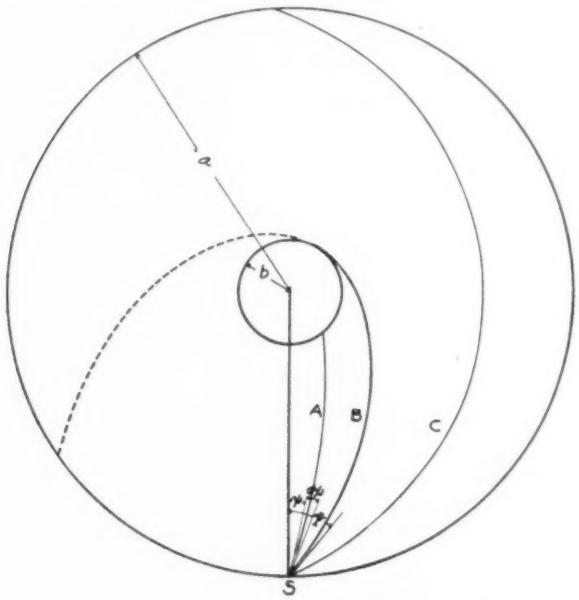


FIG. 1.—Diagrammatic central anode photo-electric cell

In other words, *the voltage current relation is different for light of different colors.*

This effect was discovered in the course of some experiments conducted with other objects in view upon a series of potassium photo-electric cells. In every cell measured it was found that the rate at which the current rose toward saturation with increasing voltage was less the greater the frequency of the light used. This phenomenon appears not to have been noticed or discussed before, although we shall find in the study which follows that it is a necessary consequence of well-recognized laws of photo-electric emission.

2. *General theoretical considerations.*—For the purposes of a theoretical study we shall assume a photo-electric cell (Fig. 1) consisting of a spherical target of radius b, at the center of a spherical shell of conducting material, the radius of which is a. On the surface of the shell we assume a small spot of photo-sensitive material S. The bulb is assumed to have a window of negligible size through which the sensitive spot may be illuminated. The central sphere is to be connected to the positive side and the shell to the negative side of a battery the voltage of which can be varied at will.

If an electron is emitted from S it will traverse a path of some form or other under the action of the electrical field which exists between the two electrodes and ultimately will either collide with the target, and thus contribute to the current observed, as in the case of curve A (Fig. 1), or else it will collide with the spherical shell as in curve C, and thus be returned to the original circuit. The problem of determining the voltage-current characteristic of the tube is therefore identical with that of determining what proportion of the total number of electrons emitted traverse paths of such a shape as to bring them into collision with the central target.

The path traveled by an electron is completely determined by two initial properties. The one of these is the velocity with which the electron is emitted; the other is its initial direction relative to the radius of the sphere. If every electron possessed the same initial velocity and was shot off at the same angle with the radius as every other electron, all would describe similar paths, and therefore either all or none would be caught by the target. Under these circumstances no current whatever would flow until the potential difference between the electrodes reached a certain critical value, at which value the saturation current would suddenly come into being and would continue unchanged for all higher voltages.

It is obvious, therefore, that the current-voltage relation which we seek depends in a very vital way upon the distribution of initial velocities and initial directions among the electrons, and it is our problem to show the effect of each of these factors.

3. *The voltage-current relation when all electrons have the same initial velocity but not the same initial direction.*—Referring again

to Figure 1 we notice that, whatever the exact form of the electron's path, there must be a certain limiting angle Ψ possessing the property that all the electrons of the same initial velocity whose initial directions lie within a cone of central angle 2Ψ about the radius as an axis contribute to the observed photo-electric current, while those which have initial directions lying outside this cone do not. If we denote by n_0 the number of electrons emitted and adopt the symbol $\phi(\psi)d\psi$ to represent the fraction having initial directions which lie between cones of central angles 2ψ and $2(\psi+d\psi)$ about the radius as an axis, it is evident that the number reaching the anode is given by the formula

$$n = n_0 \int_0^{\Psi} \phi(\psi) d\psi. \tag{1}$$

Since the limiting angle Ψ is a function of the potential difference between anode and cathode, it follows that the ratio $\frac{n}{n_0}$ is defined by (1) as a function of the voltage. Since this ratio is equal to the current if the saturation current is taken as the unit, we shall hereafter for simplicity speak of $\frac{n}{n_0}$ as "current." With this understanding, (1) is the equation of the voltage-current distribution for the case in which all electrons have like initial velocities. However, before it can be of any service to us, we must know both the relationship between Ψ and the potential difference V, and the nature of the distribution function $\phi(\psi)$.

Since the force between the two spheres is central and inversely proportional to the square of the distance from their common center, it follows that the path of an electron emitted from the shell is an ellipse having the center of the spheres as one of its foci. The curves A, B, and C of Figure 1 are drawn as such ellipses. If r and θ are polar co-ordinates referred to the focus of this ellipse as origin and its major axis as axis of co-ordinates, the equation of this ellipse is

$$r = \frac{v^2 a^2 \sin^2 \psi}{\mu + 1\sqrt{\mu^2 + (v^4 a^2 - 2\mu v^2 a)\sin^2 \psi \cos \theta}}. \tag{2}$$

In this equation v and ψ are the initial velocity and direction, and

$$\mu=\frac{\frac{e}{m}V}{\frac{1}{b}-\frac{1}{a}}.$$

Once we know this equation for the trajectory, it is a simple matter to find the relation between V and Ψ. All that is necessary is to build up the condition that the least value of r as given by (2) shall be the radius b of the anode. It is obvious that this least value of r corresponds to $\theta=0$. Hence, setting $\theta=0$ and $r=b$ in (2) and solving for Ψ, we obtain the solution

$$\sin^2\Psi=\frac{b^2}{a^2}\left[1+\frac{2eV}{mv^2}\right]. \tag{3}$$

It should be noticed that the value of Ψ defined by (3) varies with both V and v. As the voltage is increased the sine of Ψ increases (and therefore Ψ itself increases) until the voltage becomes

$$V=\frac{mv^2(a^2-b^2)}{2eb^2}.$$

For this voltage $\Psi=\frac{\pi}{2}$, i.e., all electrons which leave the cathode are caught by the anode regardless of their initial directions. For higher potential differences it is obvious from physical considerations that the current must remain unchanged, a statement which may be translated into mathematical terms by saying that for all larger values of V the upper limit of integration in (1) shall be $\frac{\pi}{2}$.

We have thus established the relationship existing between Ψ and the potential difference V. This is one of the two facts which we have already said must be available before (1) can be computed. The other is the nature of the distribution function $\phi(\psi)$. With regard to this latter it seems quite reasonable to assume that the number of electrons emitted in unit solid angle in the direction ψ is proportional to $\cos\psi$, so that the fraction emitted between cones of central angles 2ψ and $2(\psi+d\psi)$ is

$$\phi(\psi)d\psi=2\sin\psi\cos\psi\,d\psi. \tag{4}$$

Introducing this formula into (1) and integrating between the limits 0 and Ψ we find that the current $\frac{n}{n_0}$ is either

$$\frac{n}{n_0} = \sin^2 \Psi = \frac{b^2}{a^2}\left[1 + \frac{2eV}{mv^2}\right] \tag{5}$$

or

$$\frac{n}{n_0} = 1 ,$$

according as V is less than or greater than the critical voltage at which saturation first occurs. The voltage-current relation therefore takes the form shown in Figure 2. When both cathode and anode are at the same potential the current is $\frac{b^2}{a^2}$ times the saturation current, and it increases linearly with the voltage until

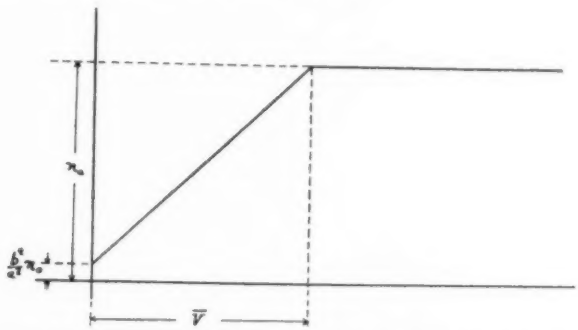


FIG. 2.—Voltage-current relation where electrons are emitted in all directions at a single velocity.

the critical potential $\bar{V}$ is reached, continuing unchanged for all higher voltages.

All of this is on the assumption that all electrons are emitted with the same initial velocity. Our next problem will be that of determining what conditions exist when this is not true.

4. *The voltage-current relation when both initial direction and initial velocity are different for different electrons.*—Let us assume that all the electrons shot off from the cathode are grouped into classes, the individual members of which have approximately the same velocity. Let us call the total number shot off n_0 and denote

by $f(v)dv$ the proportion of these which have velocities between v and $v+dv$. Each of these classes will have its own current-voltage curve similar to the one shown in Figure 2, but the curves for different classes will differ from one another in two important respects: (*a*) Since the number of electrons is not the same for all classes, the horizontal portion of the curve will not be at the same height in every case. (*b*) Since V varies with the initial velocity v, the voltage at which the break in the curve occurs will differ from one class to another. For this reason a set of voltage-current curves for the separate classes would present somewhat the appearance shown in Figure 3, where the larger subscripts indicate the

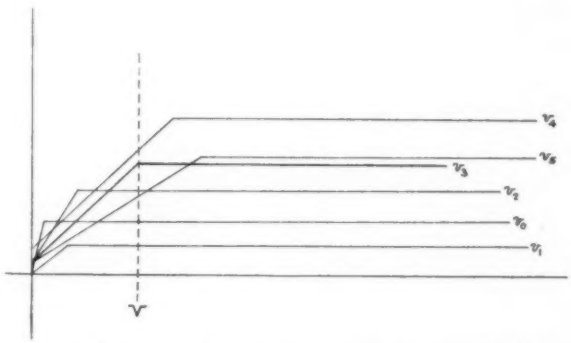


FIG. 3.—Current increments due to groups of electrons emitted in all directions at different velocities.

larger values of v. Each of these curves represents a fraction of the total current observed. The complete current for any voltage may obviously be obtained by taking the sum of all the ordinates corresponding to that voltage. Since these different ordinates correspond to different values of v this sum must of necessity take the form of an integral with respect to v. It only remains to determine the nature of the integral.

Consider the ordinates for the value of V shown in Figure 3. For the three lowest values of v, which have been called v_0, v_1, and v_2, respectively, this voltage is sufficiently high to produce the saturation current. The ordinate is therefore

$$n_0 f(v) \tag{6}$$

in each case.

For the curves v_4 and v_5 the voltage V is not large enough to produce the saturation current, and therefore the ordinate instead of being equal to $n_0 f(v)$ is smaller than this by a factor determinable from the equation of the sloping portion of the curve. Referring to equation (5) we find that under these circumstances the ordinate is

$$n_0 f(v) \frac{b^2}{a^2}\left[1+\frac{2eV}{mv^2}\right]. \tag{7}$$

For the particular velocity v_3, the break comes exactly at the voltage V, so that for this one velocity either (6) or (7) is a legitimate formula.

Since the formula for our ordinates is different according as v is less than or greater than v_3 we cannot express their sum by means of one integral, but must break up the sum into two integrals taken over appropriate ranges. The result is:

$$\frac{n}{n_0} = \int_0^{v_3} f(v)dv + \int_{v_3}^{\infty} f(v)\frac{b^2}{a^2}\left[1+\frac{2eV}{mv^2}\right]dv. \tag{8}$$

Before this formula can actually be evaluated we must know two things: (1) the relation between v_3 and the potential V, and (2) the nature of the velocity distribution function $f(v)$. The first of these is the velocity for which the critical voltage V is the particular value of V with which we are dealing. It follows immediately from the definition of V that it is

$$v_3 = \sqrt{\frac{2eb^2V}{m(a^2-b^2)}}. \tag{9}$$

The distribution of the electrons according to velocity must be obtained from the field of experimental photo-electricity. A satisfactory theoretical formula has not as yet been found for this, so that we must adopt an empirical one. As a guide to a choice of such a formula we note the experimental findings that

a) There are no electrons having energies greater than $h(\nu-\nu_0)$ where h is the quantum constant, ν is the frequency of the incident light, and ν_0 is the frequency of the longest radiation which is capable of causing emission.

b) The number of electrons having *energies* between o and $h(\nu-\nu_0)$ may be represented approximately by a curve which is symmetrical about the mean value of the energy.

For purposes of calculation we have assumed this curve of energies, symbolized by a, to be a parabola cutting the energy axis

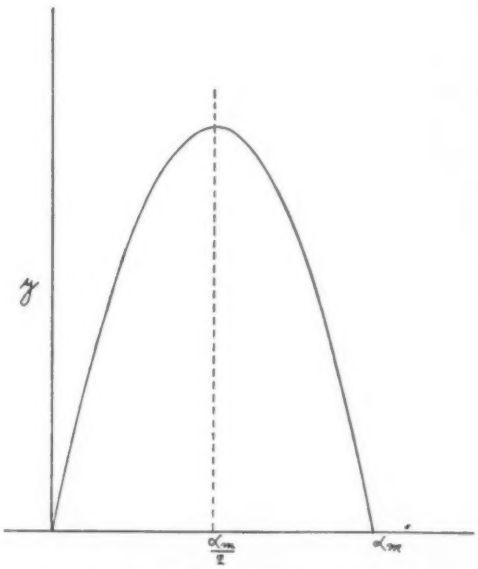


FIG. 4.—Assumed distribution of electrons according to energy of emission

at o and $a_m=h(\nu-\nu_0)$, (Fig. 4), and symmetrical about the mean energy $\frac{a_m}{2}$. The equation of this parabola is

$$y=n_0\left[1-\left\{1-\frac{2a}{a_m}\right\}^2\right], \qquad (10)$$

and therefore $y\,da$ represents the number of particles having energies between a and $a+da$.

Since energy is proportional to the square of velocity it follows that $\frac{a}{a_m}=\frac{v^2}{v_m^2}$, where v_m is the largest velocity of emission which any of the electrons can have. Substituting this value in (10) we

obtain as the number of electrons with velocities between v and $v+dv$ the function

$$n_0 f(v)dv = 4n_0\left[\frac{v^2}{v_m^2}-\frac{v^4}{v_m^4}\right]d\alpha = \frac{8n_0 a_m}{v_m^2}\left[\frac{v^2}{v_m^2}-\frac{v^4}{v_m^4}\right]v\,dv\,. \qquad (11)$$

We have now determined both of the quantities necessary to enable us to integrate (8). Carrying out these integrations we find that

$$\frac{n}{n_0}=\frac{b^2}{a^2}+\frac{a^2-b^2}{a^2}\left[3-3\frac{v_3^2}{v_m^2}+\frac{v_3^4}{v_m^4}\right]\frac{v_3^2}{v_m^2}=1-\frac{a^2-b^2}{a^2}\left[1-\frac{v_3^2}{v_m^2}\right]^3. \qquad (12)$$

This is the equation of the current-voltage curve. It shows that the current increases steadily with v_3 (which, as is easily seen from equation [9], means that the current increases steadily with voltage) until, when the value $v_3=v_m$ is reached, saturation occurs. For larger voltages it is obvious from physical conditions that the current no longer increases and that therefore $\frac{n}{n_0}=1$. The potential at which saturation occurs is easily found by equating v_3 to v_m. Since

$$a_m=\frac{mv_m^2}{2}=h(\nu-\nu_0)$$

and

$$v_3^2=\frac{2eb^2V}{m(a^2-b^2)}$$

it is

$$V_s=\frac{h(\nu-\nu_0)(a^2-b^2)}{eb^2}.$$

Also, since

$$\frac{v_3^2}{v_m^2}=\frac{V}{V_s},$$

(12) reduces to the simple form

$$\frac{n}{n_0}=1-\frac{a^2-b^2}{a^2}\left[1-\frac{V}{V_s}\right]^3. \qquad (13)$$

Computations have been made from formula (13) for $\frac{b}{a}=\frac{1}{15}$, and

for radiations of which the frequencies and wave-lengths are

$$\nu_0 = 500.0 \cdot 10^{12} \qquad (\lambda_0 = 6000 \text{ A})$$
$$\nu_1 = 545.5 \cdot 10^{12} \qquad (\lambda_1 = 5500 \text{ A})$$
$$\nu_2 = 666.7 \cdot 10^{12} \qquad (\lambda_2 = 4500 \text{ A})$$

The results of these computations are shown in Figure 5. The curves show that if the assumptions which we have made are correct we may expect the current to rise to complete saturation only at fairly high voltages—40 in the case of illumination by a

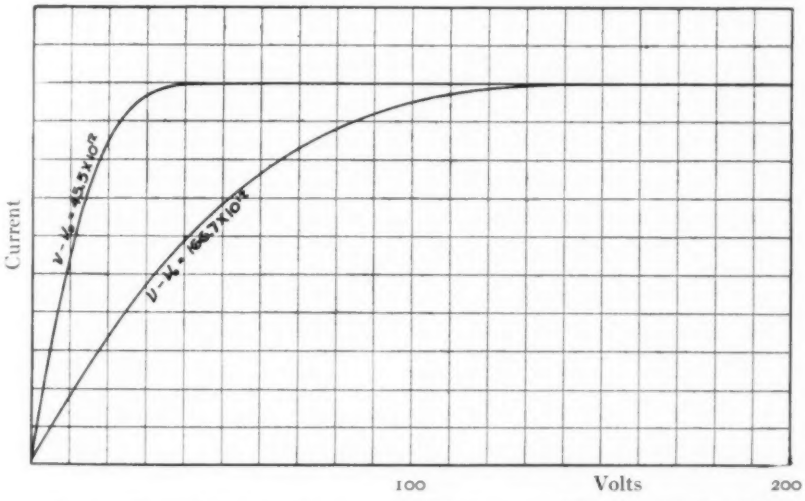


FIG. 5.—Computed voltage-current curves for two different wave-lengths of exciting radiation.

wave-length only 500 A from the limiting wave-length, 160 for a wave-length 1500 A from the same limit. If these computations are correct they mean that, in the design of cell considered, an initial difference of velocities corresponding to only half a volt results in diverting electrons from the anode which can only be recovered by an applied field of over a hundred volts. The ratio between photo-electric currents produced by two different wave-lengths may vary by a factor of as much as two, depending on whether or not the complete saturation voltage is used.

5. *Experimental confirmation.*—The curves of Figure 5 are not greatly different from those obtained with the central grid potassium

cells in which the effect was first observed. However, in order to approximate more closely to the conditions assumed in the computations, a special cell was built. This is shown diagrammatically with its dimensions in Figure 6. The outer shell was of glass and was silvered on the inside except for a small window W. The anode A was a brass ball ($b = \frac{1}{15} a$ as in the computed case) carried on a slender wire leading out through the stem, to which

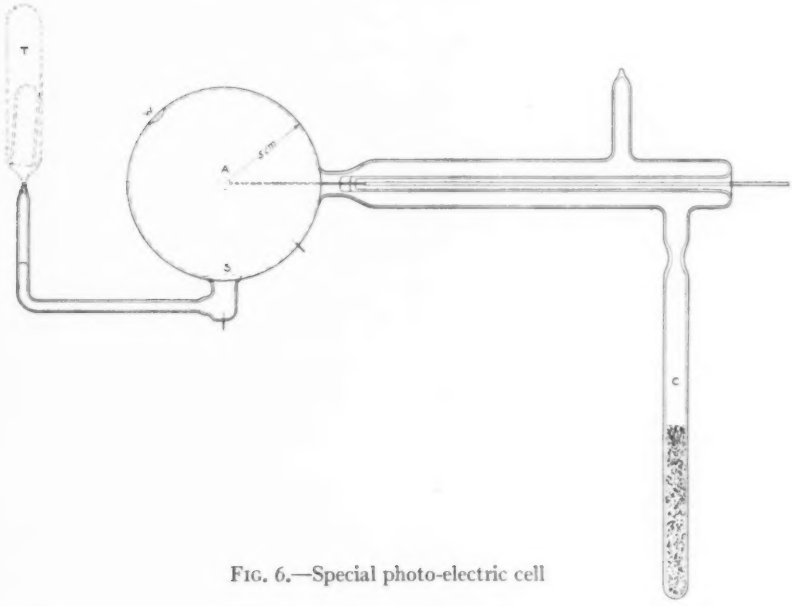


FIG. 6.—Special photo-electric cell

was connected a charcoal tube C. In preparing the cell, the whole structure was first thoroughly baked out on the pump at 350° C. Then a pyrex glass tube of vacuum-distilled potassium was introduced into the side tube T. Finally after long pumping and roasting of the charcoal in C, the potassium was melted and ran into the bulb through the connecting tube, forming a pool at S.

This cell was investigated for its voltage-current relation by illuminating the potassium with an incandescent lamp, the light from which passed through colored glasses. Currents were measured by a wall galvanometer. It was found that it was sensitive

to the whole visible spectrum, so that the limiting wave-length must lie in the infra-red. In this respect it differs from the cell assumed in the theoretical study ($\lambda_0 = 6000$), but inspection of the theoretical development shows that the actual limiting wave-length is of no special consequence, since the computations depend only upon the amount by which the wave-length of the incident light differs from this limiting wave-length.

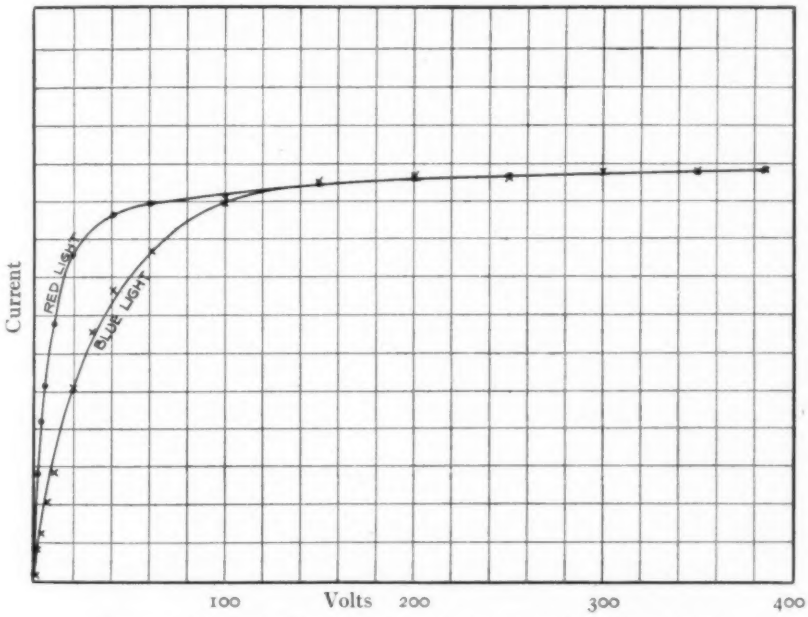


FIG. 7.—Observed voltage-current relation for red and blue light

Figure 7 shows two voltage-current curves; one taken with a monochromatic red glass, the other with a fairly monochromatic blue glass, the illumination being so adjusted that the currents at 400 volts were equal in both cases. Comparison of these curves with those of Figure 5 shows that the predictions of the theory are substantially correct. The rather close similarity in shape of the predicted and experimental curves may be taken to show that the various assumptions as to the distribution of emission with respect to direction and velocity are not seriously incorrect. However, the experimental red-blue curves lie closer together than is

to be expected from the theory, since they are separated by somewhat less than the amount computed for a yellow-blue frequency difference. This quantitative discrepancy is probably to be ascribed to some effect of residual gas, and is being investigated further.

6. *Discussion.*—Two fields suggest themselves in which this effect may influence the interpretation of measurements made

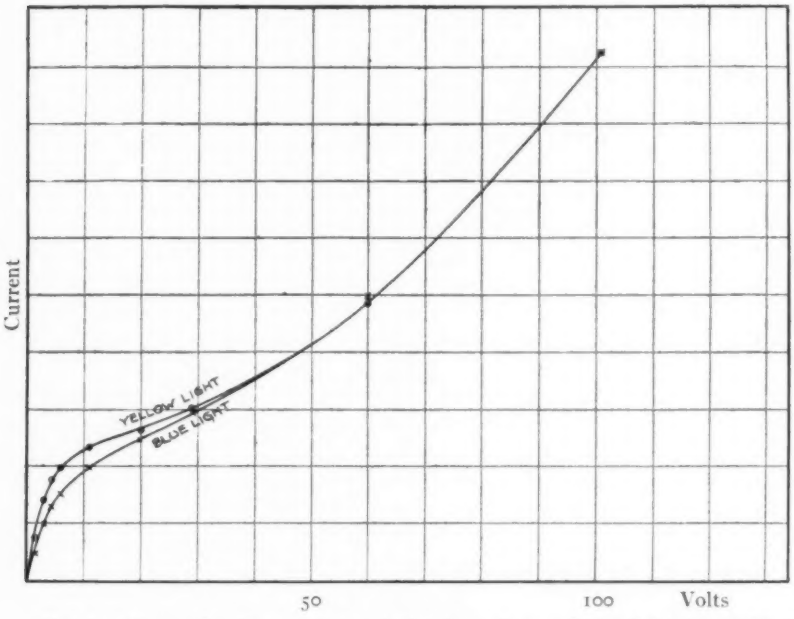


FIG. 8.—Observed voltage-current relation for yellow and blue light, in gas-filled cell.

with the photo-electric cell. One is in the determination of spectral distribution of response in connection with theories of the photo-electric effect. Unless a potential difference of several hundred volts is used (where the frequencies utilized cover a range as great as the visible spectrum), the resultant curves connecting wave-length and response will be distorted. A second field is in the use of photo-electric cells for investigating color relations, such as the color indices of stars. For any such application the cell must obviously be calibrated and used at one definite voltage,

unless the voltage employed produces complete saturation. In this connection a pertinent question is whether these differences in the current relation are present in a gas-filled cell. The answer is that in general they are, as is shown by Figure 8, presenting data obtained with a gas-filled cell with a grid anode closely similar to those which have been used in stellar photometry. However, owing to other factors which may be discussed in a future communication, the presence of the gas frequently acts in such a way as to diminish the effect here described.

RESEARCH LABORATORIES OF THE AMERICAN
TELEPHONE AND TELEGRAPH CO., AND THE
WESTERN ELECTRIC COMPANY, INC.
March 14, 1922

THE WIDTH OF CERTAIN LINES OF THE SPECTRUM OF HELIUM AS A FUNCTION OF PRESSURE IN THE SOURCE

By LLOYD W. TAYLOR

ABSTRACT

Visibility-curve photometer for measuring the width of spectrum lines.—In the visibility-curve method, the difference of path of the two beams of light in a Michelson interferometer is increased until the visibility of the circular fringes drops to half its maximum value. To enable the visibility as a function of path difference to be determined more accurately, a comparison system of circular fringes of known and variable visibility was provided by means of a concave plate of doubly refracting crystal (selenite) placed in a beam of monochromatic plane-polarized light and viewed through a nicol prism. The two systems of fringes appeared side by side, and readings were taken by rotating the crystal plate about its axis of symmetry until both systems were equally visible. In this way line widths of from 0.03 to 0.12 A were measured within a few per cent.

Widths of three helium lines as a function of pressure.—Pure helium was excited in an ordinary Plücker tube by means of a transformer. *The effect of varying the power input* was found to be small, in fact multiplying the power by $4\frac{1}{2}$ increased the width only 12 per cent. Also a change of capillary diameter from 1 mm to 2 mm did not affect the results. Using a constant power of about 9 watts, the widths of the *lines* λλ *6678, 5876, and 5016* were measured to pressures of 12, 35, and 22 cm respectively. Contrary to expectation, the widths do not increase quite linearly with the pressure but approximately according to the formulae: $2W(6678) = .045 + .0088p - .0002p^2$; $2W(5876,\ \text{main component}) = .040 + .0034p - .00012p^2$; $2W(5016) = .030 + .0045p - .0001p^2$; where p is in centimeters of mercury and $2W$ is in Angstroms.

INTRODUCTION

That the spectral lines into which the radiation emitted by incandescent gases or vapors may be resolved possess finite and measurable widths constitutes one of the most elementary considerations in the field of spectroscopy. It is almost equally well known that the width of any one of these lines is not constant, but varies with changing conditions. One of the factors thus affecting the width of spectral lines is the pressure which exists in the source.

The present investigation was undertaken for the purpose of discovering the relation which exists between pressure and width in some of the more prominent lines of the spectrum of helium. Three of these lines (λλ 5016, 5876, and 6678 respectively) have been studied. A preliminary report on one of them (λ 5876)

was read before the American Physical Society on November 27, 1921.[1] The present article extends the somewhat limited conclusions that it was possible to make at that time concerning that line and makes a similar report on the other two named.

HISTORY

The first observation on record concerning the effect of pressure on spectral line width is that of Ångström[2] in 1853. Among other early observers may be mentioned Wüllner[3] and Frankland.[4] In 1889 Rayleigh[5] investigated theoretically molecular velocities in the line of sight (the Doppler effect) as a possible cause of the width of spectral lines. This is of course a temperature phenomenon. In two subsequent contributions, Michelson[6] extended Rayleigh's theory to include pressure effect, and at the same time published the first comprehensive experimental data. His observations on the pressure effect were made on the α line of hydrogen at pressures up to 9 cm of mercury, and on prominent lines in the spectra of a number of metallic vapors. His theory indicated linear increase of width with pressure, and his observations indicated that the theory furnished "a very good first approximation to the truth," since he had observed that in all cases investigated "when the pressure is increased, the width increases in a nearly linear proportion."

Rather steady contributions to this field were made during the twenty years subsequent to this work. All of this, as well as the earlier work, has been summarized by Stark,[7] who in a comprehensive publication outlines the status of our present knowledge of the width of spectral lines, and the factors affecting it. The results of several contributions, both theoretical and experimental, have been published since then. It is probable, however, unless

[1] Taylor, *Physical Review*, **19**, 255, 1921.

[2] Ångström, *Annalen der Physik*, **94**, 141, 1855.

[3] Wüllner, *Annalen der Physik*, **137**, 339, 1869.

[4] Frankland, *Proceedings Royal Society*, **17**, 288, 1869.

[5] Rayleigh, *Nature*, **8**, 474, 1873.

[6] Michelson, *Philosophical Magazine*, **34**, 280, 1892; *Astrophysical Journal*, **2**, 251, 1895.

[7] Stark, *Jahrbuch der Radioaktivität*, **12**, 349, 1915.

some revolutionary discovery should be made, that Stark's publication will constitute the chief reference for some time to come concerning the facts and theories of the width of spectral lines.

PRELIMINARY CONSIDERATIONS

There are a number of factors known to influence the width of spectral lines. These may be classified in different ways, depending on the point of view. It will be convenient for our purpose to divide them into three groups: temperature, pressure, and electrical effects respectively. A number of experimental studies have been made recently on these factors, among which should be mentioned Buisson and Fabry[1] on temperature effect in some of the rare gases, Rossi[2] on the effect of high pressure in the hydrogen arc, and Merton[3] on electrical effects in helium. In general, there is less experimental material on the effect of pressure than on either of the other two. It is hoped that the present investigation may make a step toward supplying this deficiency.

There were several considerations pointing toward the advisability of using helium as the gas to be studied. One was the fact that under discharge-tube conditions its visible spectrum consists of a relatively few quite brilliant lines, sufficiently separated so that the present method of investigation was readily applicable. The second consideration was the familiar fact that helium will maintain the vacuum-tube type of discharge up to higher pressures than will any other gas.

Another consideration was its low atomic weight. There is considerable evidence to show that, other conditions being fixed, the width of a line is inversely proportional to the atomic or molecular weight of the gas constituting the source. There is some advantage therefore, particularly in the earlier stages of an experimental study, in choosing a gas of low atomic weight. And finally, its chemical inertness and the supposedly simple structure of its atom furnish every reason for accumulating all the data concerning its physical properties that can in any way be made available.

[1] Buisson and Fabry, *Journal de physique*, **2**, 442, 1912; *Comptes rendus*, **154**, 1349, 1912.

[2] Rossi, *Astrophysical Journal*, **34**, 299, 1911.

[3] Merton, *Proceedings Royal Society*, *A* **95**, 30, 1918.

METHOD

In carrying out the experiment, the interference method known as that of visibility-curves has been chosen,[1] and the half-width of the lines as defined by the fall of visibility to one-half its maximum with increasing difference of optical path has been measured for different pressures. Two other methods were available: (1) direct measurement of width in a high-order spectrum such as that given by an echelon grating and (2) the interference method involving observation of the point at which the fringes in a Fabry-Perot interferometer cease to be visible with increasing distance between the mirrors. It is believed, however, that the visibility-curve method avoids the blunders which would be inevitable in the direct measurement of a length so small as that involved in the width of spectral lines, and also the high element of personal equation which is necessarily involved in any limit-of-perception method. The visibility method possesses the significant advantage that any maladjustment of the apparatus, or any shortcomings of the observer become immediately apparent, and the observer is furnished definite grounds for rejecting his observation.

The manipulative problem involved in the investigation was twofold: (1) the purification and control of pressure of the helium and (2) the taking of visibility data.

It is not considered necessary to enter into any detailed description of the purification system. The familiar method of passing the gas over hot copper oxide to remove hydrogen, and of absorbing other impurities by charcoal in liquid air, was used. A spectroscopic trace of neon existed in the purified gas at the higher pressures, and sufficient hydrogen to become just visible under a disruptive discharge, but not otherwise. The helium used as a source of supply came from the natural-gas fields of the Southwest, and was initially about 97 per cent pure. A Toepler pump was used for the double purpose of circulating the gas over the battery of purification tubes and of controlling the pressure of the purified helium in the discharge tube.

The optical part of the apparatus is shown in Figure 1. Light from a discharge tube is passed through a condensing lens into a

[1] Rayleigh, *Philosophical Magazine*, **27**, 298, 1889.

Michelson interferometer, which is adjusted for circular fringes. The emergent light is focused on to the slit of a prism spectrometer. On the images of the slit (whose aperture is maintained as wide as circumstances will permit) are seen the circular fringes from the interferometer. The line to be investigated is brought into the field of view of the telescope, and the visibility of the fringes measured, over as great a difference of path as they can be seen.

Direct estimation of the visibility of a set of interference fringes, except by an eye which has been exceedingly carefully trained, is a process whose precision is very questionable. It

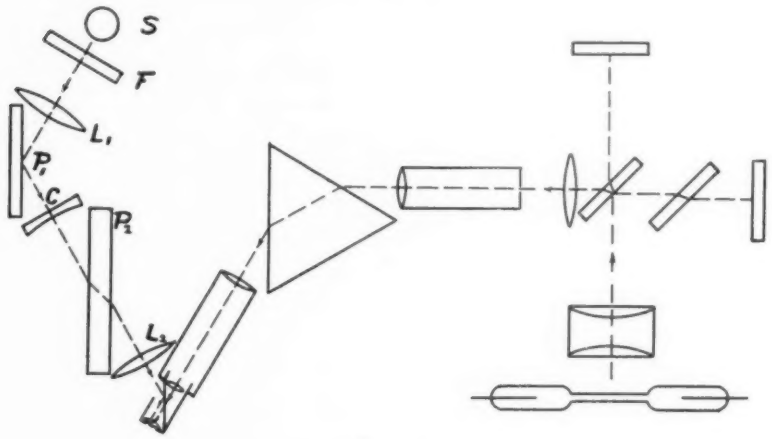


FIG. 1

has accordingly been deemed advisable for the present investigation to construct what might appropriately be termed a "visibility photometer." It is well known that if a concave plate of doubly refracting crystal be placed in a beam of monochromatic, plane-polarized light and viewed through a nicol prism, a system of concentric rings will be seen, similar in appearance to the circular fringes just described. The "visibility" of these rings can be altered at will by rotating the crystal plate about its axis of symmetry, and is a relatively simple function of the angle between the optic axis of the crystal and the plane of polarization of the incident light. If such a system be projected into the field of view of the observing telescope, the observer sees side by side the

two systems of circular fringes; one from the interferometer, having an unknown visibility, and the other from the comparison source, having a known and controllable visibility. If now the comparison fringes be caused to match the interferometer fringes and the orientation of the crystal noted, we have a direct measurement of visibility, possessing the degree of precision characteristic of any photometer measurement.

The application of this method is indicated in Figure 1. Light from an incandescent electric lamp S, screened by a filter F to a close color match with the line under observation, was reflected at the polarizing angle from a glass plate P_1, passed through the crystal C, and projected into the field of view of the telescope as shown. The second reflecting plate, which was mounted in the eyepiece, served as the analyzer. Collimating and telescopic lenses L_1 and L_2, and a glass-plate pile P_2, for initially matching the maximum visibility of the comparison system with that of the interferometer system, completed the equipment.

The source of illumination for the spectral lines studied was a Plücker tube of conventional type. Several different tubes were used in the course of the investigation. The only difference in their construction which could be of any conceivable importance in affecting line width was the size of the capillary portion. This was of 2 mm diameter in case of the study on λ 5876, and 1 mm in the case of the other two. Since there is a little evidence[1] that current density affects line width, several readings were taken on λ 5876 with a 1-mm tube as a source, other conditions being maintained the same as in earlier readings on the 2-mm tube. Line widths were found the same within the limits of experimental error.

The tube was excited from the secondary of a transformer energized by a 60-cycle alternating current. The potential difference necessary to excite a sufficient degree of luminescence varied from 400 to 1300 volts, and the average power consumption was in the neighborhood of 9 watts. This was sufficient to cause the tube to become almost dangerously hot, but was found necessary in order to give lines sufficiently strong for study at the higher pressures.

[1] Rossi, *Astrophysical Journal*, **40**, 232, 1914.

Since it is known that line width depends upon temperature and electrical conditions (e.g., upon capacity and induction in the circuit) as well as upon pressure it was necessary to take precautions to maintain both of these conditions constant through the course of the investigation. Constancy of electrical conditions, aside from current and potential-difference, was easily maintained by measures so elementary that they do not need to be recounted. Preliminary experiments on temperature influence indicated that the conditions which must be satisfied in order to insure that the effect of temperature changes was negligible in comparison with the effect of pressure were not particularly exacting. The evidence was conclusive that temperature of a sufficient degree of constancy could be maintained by maintaining in the tube a constant power consumption. Hence the current was modified in inverse proportion to the potential-difference found necessary to drive a discharge through the tube at different pressures, and the assumption was made that constant power consumption satisfied sufficiently the condition of constant temperature. Needless to say, sufficient time was always allowed for the tube to reach a state of radiation equilibrium with its surroundings before any readings were taken.

For the purpose of determining roughly the effect of large changes in power consumption, visibility-curves have been run under different power consumptions. It was found that it was necessary to alter it through 100 per cent before the resulting change in width became at all measurable. With the consumption raised to four and one-half times its initial value, the increase in half-width of the lines became of the order of 12 per cent. Since during the course of this investigation the power consumption underwent a maximum variation of less than 20 per cent, it was considered that we were well within the limits of precision imposed by constant temperature.

Half-widths (W) were calculated from observations of the difference of path (Δ) necessary to cause a fall of visibility to half its maximum, in accordance with the familiar relation

$$W = \frac{.22}{\Delta}\lambda^2.$$

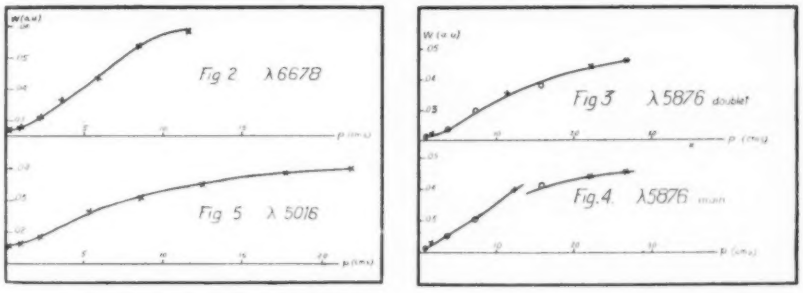
As is well known, this relation is derived on the assumption that the distribution of intensity within the line follows an exponential law of the form

$$f(x) = e^{-k^2x^2},$$

where $f(x)$ represents intensity, x represents the distance on either side of the central maximum, and k is a constant. The "half-width," W, corresponds to a value $f(x) = 1/2$, and is hence the distance from the *central maximum* to the point of half-intensity on either side. It is quite possible to adopt any other measure of the line width, provided that this be sufficiently defined. The one here given is, however, the convention which meets with general acceptance.

RESULTS

The results of the present investigation are summarized in Tables I, II, III, and IV, and are presented in graphical form in



FIGS. 2, 3, 4, 5

Figures 2, 3, 4, and 5. Each plotted ordinate on these curves represents the mean half-width as determined from measurements on from four to twelve visibility-curves taken on that line at that pressure.[1] This is not, of course, a sufficient number of observations to warrant any conclusion as to the "probable error." Aside from completeness in description of apparatus and procedure, the best criterion for the magnitude of the experimental error would probably be given by a publication of all of the visibility-curves

[1] The conditions surrounding the highest-pressure readings on λ 5876 could not be duplicated, and since only one reading was possible, the degree of precision of the measurement of width at that pressure may be questioned. The reading is submitted, however, for what it is worth.

taken. Since this is scarcely feasible, the following data concerning deviations of values of the half-width from the mean value are given. It is to be noted that the closeness of agreement between different readings on the same line under the same conditions depends, in this type of investigation, mainly on the sensitivity of the eye to photometric matches of intensity, and that this depends largely on color. There is consequently some justification in grouping deviations from mean values of half-width separately for each line.

Per cent of deviation from mean		λ 6678	λ 5876	λ 5016
	maximum	3.9	6.8	5.4
	mean	1.7	3.0	2.2

TABLE I

6678 (1*P*) − (*mD*)

Pressure (cm)	Δ (mm)	*W* (a.u.)
11.8	16.8	.0584
8.4	18.2	.0539
5.9	22.5	.0436
3.5	26.6	.0369
2.2	31.5	.0312
0.9	35.2	.0279
0.2	36.4	.0220

TABLE II

5876 (1*p*) − (*md*)

Pressure (cm)	Δ (mm)	*W* (a.u.)
35.3	14.0	.0543
26.9	16.3	.0460
22.3	17.2	.0442
15.8	20.0	.0380
11.3	21.5	.0354
7.3	25.5	.0298
3.6	32.1	.0237
1.6	34.0	.0224
0.8	35.9	.0212

TABLE III

5876 (MAIN)

Pressure (cm)	Δ (mm)	*W* (a.u.)
35.3	14.0	.0543
26.9	16.3	.0460
22.3	17.2	.0442
15.8	18.4	.0413
11.3	19.0	.0400
7.3	24.9	.0305
3.6	30.0	.0253
1.6	32.6	.0233
0.8	35.7	.0213

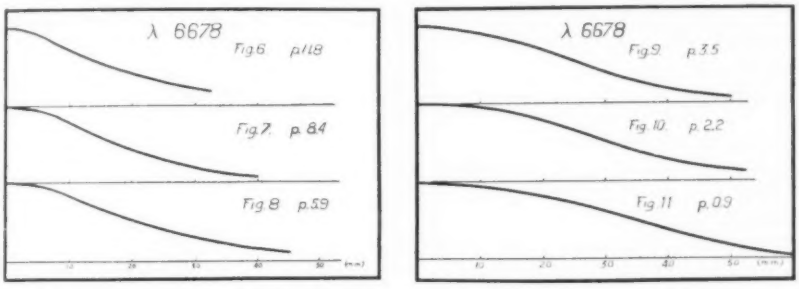
TABLE IV

5016 (1*S*) − (*mP*)

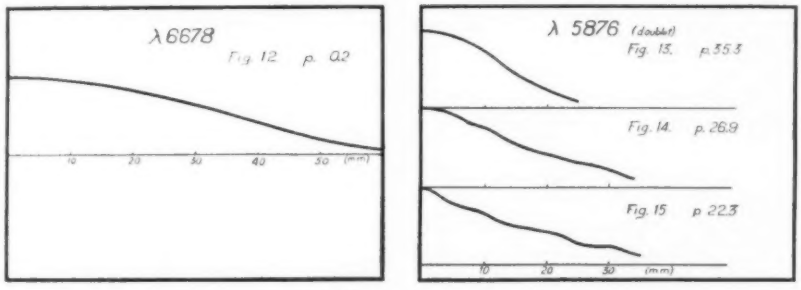
Pressure (cm)	Δ (mm)	*W* (a.u.)
22.0	13.8	.0401
17.8	14.2	.0389
12.5	15.7	.0352
8.6	18.0	.0308
5.3	20.6	.0269
2.2	30.7	.0181
0.9	33.8	.0164
0.2	35.4	.0157

There will be found in Figures 6–35 one visibility-curve for each line at each pressure. Each of these curves is the graphical "average" of all visibility-curves taken on that line at that pres-

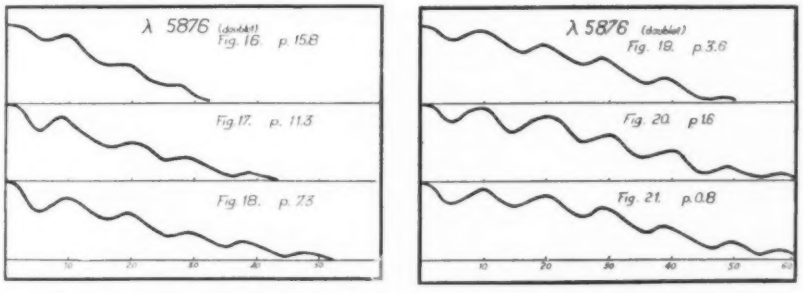
sure. As will be noted, the visibility-curves for λ 5876 represent a double source. In this case, the half-width is by common convention defined by the fall of the *envelope* of the visibility-curve to



FIGS. 6, 7, 8, 9, 10, 11



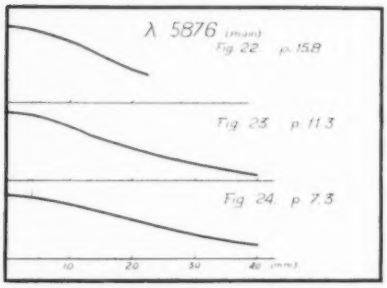
FIGS. 12, 13, 14, 15

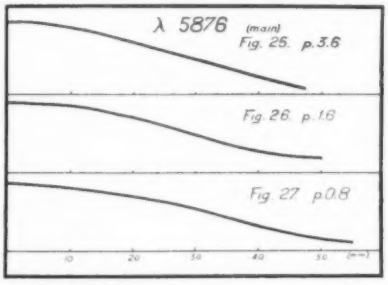


FIGS. 16, 17, 18, 19, 20, 21

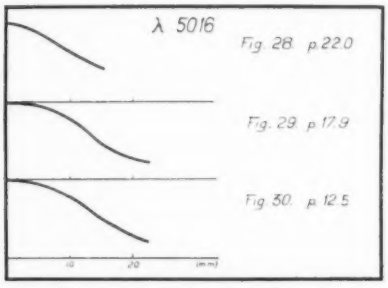
the ordinate representing half-visibility. The curves of Figures 22–27 do not constitute an independent set of readings, but are analyzed out of the curves of Figures 16–21 by methods which are

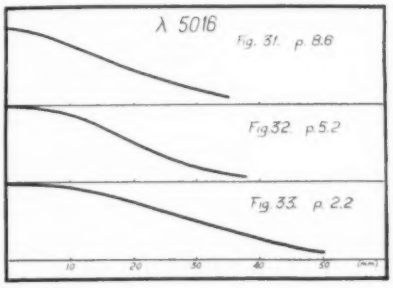
immaterial here.[1] The minor components of λ 5876 are too faint to allow of any conclusive inference concerning its half-width.



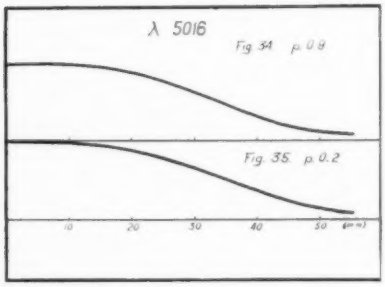


FIGS. 22, 23, 24, 25, 26, 27





FIGS. 28, 29, 30, 31, 32, 33



FIGS. 34–35

With increasing pressures, the intensity of the lines becomes continually lower. The point at which the intensity of any line became so low that accurate judgment of visibilities became impossible, determined the upper limit of pressure at which readings could be taken.

[1] With increasing pressure, the visibility-curve for the λ 5876 changed its character, possessing less the features of a doublet and more nearly those of a single source as the components merge into one another with increasing width. Above a pressure of 15.8 cm it became impossible to analyze out the visibility-curve for the main component, and from this point on no distinction was made between the width of the doublet and that of the mean component. This causes a discontinuity in the pressure-width curve (Fig. 4).

As will be noticed, the order in which this limit was reached with increasing pressure was λλ 6678, 5016, and 5876 respectively.

It will be noted that Figures 2–5 show that half-width is an increasing function of pressure, but *not* a linear function. The deviation from linearity is sufficiently small so that one could proceed over considerable ranges of pressure in properly chosen portions of the curves without any very startling deviation from a linear variation of half-width. In fact, on the occasion of the author's preliminary report, referred to above, at a time when the only information available was given by the four points designated with circles, the conclusion was drawn that "the half-width of the line is found to increase at a rate which is very nearly linear," and the statement was made that an attempt was under way to "materially extend the upper limit of pressure, in order to discover whether, at higher pressures, there may exist any notable deviation from this law."

Such a deviation seems to exist, under the conditions of this experiment. It is not of any very great magnitude, but it is too consistent, and too far outside of what may be considered the limits of experimental error, to be ignored. All available information on the theory of pressure-broadening, with one exception,[1] indicates a linear broadening with increase of pressure, a prediction which up to the present seems to have been confirmed by experiment. The discrepancy between these considerations and the observations submitted herewith has been the occasion for an exceedingly careful scrutiny of the work performed. Failure to find any circumstances outside of a real pressure-effect which might form a cause, has rendered advisable a rather minute description of the conditions under which the experiment was performed.

It should perhaps be remarked that the method used in this investigation does not permit the detection of any lack of symmetry in the broadening of the lines with increasing pressure. There is considerable evidence to show that, at least at the pressures used, no such asymmetry is to be expected. If it should occur, it would probably manifest itself in an apparent change of wave-length with changing pressure. An investigation is at present under

[1] Gianfranceschi, *Il Nuovo Cimento*, **18**, 57, 1919.

way in this laboratory for the purpose of detecting whether any such "pressure-shift" is to be observed.

SUMMARY

The widths of three of the prominent lines in the spectrum of helium have been measured for different pressures in the source. "Half-widths" have been found to increase with increasing pressures, at rates which exhibit consistent deviations from the linear one called for by the theory.

The method used was that of visibility-curve analysis.

The writer takes pleasure in acknowledging his indebtedness to Professor H. G. Gale, who suggested both the problem and the method, and who has maintained a constant interest throughout the course of the investigation; to Professor A. A. Michelson for the use of the interferometer and concave selenite plate which he himself had used in previous investigations; and to Mr. F. M. Kannenstine for valuable suggestions concerning the purification of the helium.

RYERSON PHYSICAL LABORATORY
UNIVERSITY OF CHICAGO
March 10, 1922

ON EINSTEIN'S ABERRATION EXPERIMENT

By C. V. RAMAN

ABSTRACT

Aberration experiment proposed by Einstein to decide between the theories of light.—According to Einstein, if light from swift canal rays were focused on a slit and the transmitted rays were made parallel and were then focused on cross hairs, the image of the slit would be shifted when a plane-parallel layer of a dispersing medium such as CS_2 is placed in the path of the parallel rays, because, due to the motion of the canal rays, the light rays on one side of the beam would have a shorter wave-length than those on the other side and hence, if the ordinary wave theory is correct, the wave front should be rotated. It is here shown, however, that this conclusion is based on a misconception of the behavior of such wave fronts; that the position of the final image is independent of the relative wave-lengths of the different light rays; and that, therefore, no shift is to be expected either according to the wave theory or according to any other theory of light. Hence the experiment cannot help discriminate between the rival theories.

In a recently published communication[1] Einstein has suggested an experiment which according to him furnishes a method for deciding between the two alternative hypotheses regarding the nature of the elementary process of light-emission now holding the field, viz., the classical or undulatory theory in which light-waves are regarded as given out continuously from the luminous atom, and the quantum theory in which light is emitted as the result of an explosive act involving a definite energy-change in the atom and uniquely determining its frequency. It is proposed here briefly to summarize Einstein's paper and then to examine his arguments. It will be shown that his discussion of the phenomena to be expected is at fault and that in reality no effect is indicated by either theory and hence the proposed experiment will not serve to differentiate between them.

The proposed experiment is based upon J. Stark's well-known observation that the light emitted by the luminous atoms in the *Kanalstrahlen* exhibits the Doppler effect, the magnitude of the shift of wave-length depending on the angle between the line of movement of the atoms and the direction of observation. Einstein points out that according to the classical or undulatory theory, light is emitted from the atom simultaneously in different directions

[1] *Sitzung. Preuss., Akad. Wiss.*, 1921, p. 882.

with different frequencies, and proposes to test whether this is actually the case in the following way:

In Figure 1, K represents a stream of canal rays, L_1 is a lens which focuses an image of the same on a slit S contained in an opaque screen. The light passing through the slit falls on a collimating lens L_2, which renders the beam parallel. The collimated beam then falls upon the object-glass of a telescope focused for infinity (not shown in the figure). An image of the slit S will then be seen in the field of view. Einstein puts forward the proposition that if now a plane-parallel layer of a dispersing medium, e.g., carbon disulphide, be put in between the lens L_2 and the observing telescope, a displacement of the image of the slit is

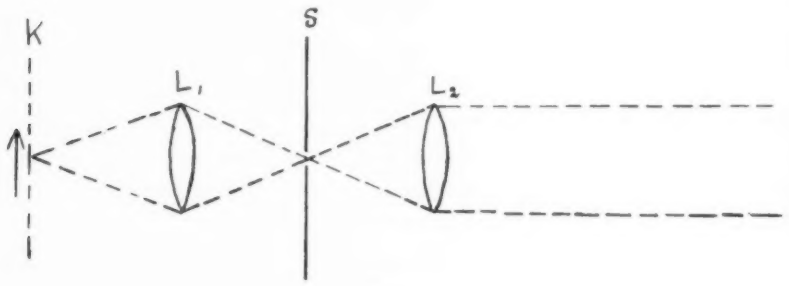


FIG. 1

indicated by the principles of the wave theory, the deviation angle α being given by the formula:

$$\alpha = \frac{l}{\Delta} \cdot \frac{v}{c} \cdot \frac{dn}{\left(\frac{d\nu}{\nu}\right)} \qquad (1)$$

where $\frac{v}{c}$ is the ratio of the velocity of the atoms in the *Kanalstrahlen* to the velocity of light, l is the thickness and n the refractive index of the dispersing medium, Δ is the distance SL_2; KL_1 and L_1S are assumed to be equal and ν is the frequency of the light.

We now proceed to indicate the argument on which Einstein bases his derivation of formula (1) which is briefly as follows: The surfaces of constant phase in the light from a luminous atom

on emergence from the lens L_2 are obviously plane. But since the light falling upon the upper and lower edges of the lens system is according to the Doppler principle of different wave-lengths, the wave fronts after emergence from L_2 are not exactly parallel to each other but are slightly inclined in the manner of the leaves of a fan. If no dispersing medium be interposed, the wave fronts after passing through the object-glass of the observing telescope reach foci, which are not coincident but form an image of the slit of finite width in the same position as when the luminous atoms emitting the waves are at rest. If, however, between the lens L_2 and the observing telescope a plane layer of a dispersing medium be interposed, the conditions are altered. The upper and lower portions of the wave fronts correspond to different frequencies and therefore move forward with different velocities and hence the wave fronts swing round to a greater and greater extent proportionately with the length of the medium traversed. Einstein then supposes that as the result of this swinging round of the wave fronts, the image of the slit in the observing telescope should suffer a corresponding deflection the magnitude of which is given by formula (1).

The error in Einstein's reasoning enters at the last stage, that is, in his assuming that the swinging round of the wave fronts involves a deflection of the image of the slit in the focal plane of the observing telescope. Referring to Figure 1, we see that as a luminous atom moves along the stream of *Kanalstrahlen*, the waves emitted by it enter the slit S only for a limited period of time depending on its width, that is, the disturbance passing through it forms a wave-group of limited extension. It is perfectly true, as pointed out by Einstein, that when the disturbance enters a dispersing medium, the wave fronts in it swing round steadily as they pass through it. But the position of the image of the slit is determined not by the inclination of any particular wave front to the axis of the optical system, but by the normal to the wave group considered as a whole. It is easily shown that the latter remains fixed and does not turn round. The problem is exactly similar to that which arises in connection with Michelson's determination of the velocity of light in dispersive media by the revolving mirror

method, and was very clearly dealt with by Willard Gibbs.[1] The group velocity is given by the relation

$$U = V - \lambda \frac{dV}{d\lambda}; \qquad (2)$$

the individual wave planes in the group rotate with an angular velocity

$$\frac{dV}{d\lambda} \cdot \beta$$

where β is the angle between successive wave planes; they also move forward through the group with a velocity $(V-U)$, and in the interval of time

$$\frac{\lambda}{(V-U)}$$

in which each wave plane moves into the position of the next in the group, it rotates through an angle

$$\beta \cdot \frac{dV}{d\lambda} \cdot \frac{\lambda}{(V-U)}$$

which by relation (2) is exactly β. Hence the second wave plane in passing a point moving with the group has exactly the same position which the first one had. Hence as the wave planes move out of the group and disappear and are followed by fresh ones, the inclination of the wave normal to the group considered as a whole remains fixed. In other words, after emerging from the layer of dispersing medium, the group would come to a focus in precisely the same position as when no dispersing medium is introduced.

The matter may also be viewed from another standpoint. Referring to Figure 1, it is clear that what is observed in the field of view of the telescope are not the moving luminous atoms but the fixed slit illuminated by the finite width of the pencil of canal rays. An appropriate method of treating the problem is therefore to regard the slit as a secondary source of light in accordance with the Fresnel-Huyghens principle. If the slit S be very narrow it may be simply regarded as a source of cylindrical waves which diverge

[1] *Nature*, April 1886, p. 582, and *Scientific Papers*, 1, 253.

from it in all directions, and in the disturbance reaching the lens L_2, all differences in the frequencies at different points would cease to exist and the image of the slit would be formed in the focal plane of the observing telescope according to the ordinary laws of geometrical optics. No difference would therefore be made by the interposition of a plane-parallel layer of dispersing medium. If the slit S be wider, we may conceive it to be divided up into a large number of very small elements, the image of each of which is formed in a fixed position in the same way as in the case of a very narrow slit, and precisely identical results are reached.

That no displacement of the image in the slit in the observing telescope can occur on the introduction of a dispersing layer can also be shown on purely kinematical reasoning independent of any theory or hypothesis regarding the nature of light. To do this, we have merely to replace the stream of canal rays at K by a stationary source which emits light of different color in different directions, e.g., a small dispersing prism from which a spectrum diverges and falls upon the lens L_1 and which after being focused at the slit S and passing through the lens L_2 is viewed through the observing telescope. It is clear that the introduction of a dispersing layer bounded by parallel faces will not influence the observed position of the source in the field of view.

CALCUTTA
March 20, 1922

THE QUANTUM THEORY OF THE SPECTRUM OF HYDROGEN

By H. A. WILSON

ABSTRACT

Energy of an electron moving near a positive nucleus.—It is shown that, at least when the speed of the electron is small, an appreciable fraction of the mutual energy of the two charges lies near the electron and therefore may be expected to move with the electron and modify its motion as determined by the quantum equation. Hence the *influence of the field of the nucleus* is probably not negligible, as is ordinarily supposed.

Quantum theory of the doublets of hydrogen and of ionized helium.—Assuming a fraction $(f/2)$ of the mutual energy of electron and nucleus moves with the electron, equations are derived for the energy in different orbits, which give Sommerfeld's expression for the separation of the doublets but multiplied by the factor $(1-f)$. Comparison of the observed separation of the hydrogen doublets (0.30) with that computed by Sommerfeld (0.365) indicates that f is about 0.2; that is, about one-tenth of the mutual energy may be moving with the electron. In the case of ionized helium, the agreement between calculation and observation indicates that f must be quite small.

In the quantum theory of the spectrum of atomic hydrogen, as developed by Bohr and Sommerfeld, it is assumed that the energy of an electron moving with a velocity v is equal to $m(1-\beta^2)^{-\frac{1}{2}}$ where m denotes its energy when at rest and $\beta=v/c$ where c denotes the velocity of light.

This expression for the energy of a moving electron follows from the electromagnetic theory of the motion of electrical systems and can also be deduced from the special theory of relativity. It is obtained in the case of an electron moving with uniform velocity in a straight line when no other charged bodies are present.

In the hydrogen atom the electron is supposed to be describing an orbit round the positive nucleus so that it is not clear that its energy can be taken equal to $m(1-\beta^2)^{-\frac{1}{2}}$ as though it were moving uniformly along a straight line.

Let the electron be a hollow sphere of electricity of amount e and radius a and let the positive nucleus be another one of amount E and radius b and let the distance between their centers be r. The electrostatic energy of the electron and nucleus when they are at rest is then

$$\tfrac{1}{2}\frac{e^2}{a}+\tfrac{1}{2}\frac{E^2}{b}+\frac{eE}{r}$$

the total energy is

$$\tfrac{2}{3}\frac{e^2}{a}+\tfrac{2}{3}\frac{E^2}{b}+\frac{eE}{r}$$

because the electron and nucleus must be supposed to contain internal energies $\frac{1}{6}e^2/a$ and $\frac{1}{6}E^2/b$ respectively.

If the electron and nucleus are supposed to be both moving with uniform velocity v in the same direction the total energy will be

$$\left(\tfrac{2}{3}\frac{e^2}{a}+\tfrac{2}{3}\frac{E^2}{b}+\frac{eE}{r}\right)(1-\beta^2)^{-\frac{1}{2}}$$

in accordance with the general principle that the energy of a system moving with uniform velocity in a straight line is equal to its energy when at rest multiplied by $(1-\beta^2)^{-\frac{1}{2}}$.

The energy $m(1-\beta^2)^{-\frac{1}{2}}$ of the electron, where $m=\frac{2}{3}e^2/a$, is located very near to its center because the energy density varies inversely as the fourth power of the distance, so that it is clear that this energy can be regarded as moving with the electron.

The energy $eEr^{-1}(1-\beta^2)^{-\frac{1}{2}}$ is distributed symmetrically about the nucleus and electron, when they are moving slowly, but is not concentrated near to them so that it is not clear that it can be regarded as associated with either of them although when both have the same velocity it certainly moves along with them.

If the nucleus is supposed fixed and the electron moving near it, then the energy $m(1-\beta^2)^{-\frac{1}{2}}$ is regarded as moving with the electron and the energy $2l(1-\beta^2)^{-\frac{1}{2}}$, where $l=eE/2r$, is usually regarded as remaining at rest. This makes the total energy equal to

$$m(1-\beta^2)^{-\frac{1}{2}}+2l+\tfrac{2}{3}\frac{E^2}{b},$$

so that if $\mathfrak{E}$ denotes the lost energy, or the difference between the energy when the electron and nucleus are at rest at a great distance apart and the energy in the orbit the electron is describing, then

$$\mathfrak{E}=m-m(1-\beta^2)^{-\frac{1}{2}}-2l,$$

which is the energy equation used in the quantum theory.

This equation is obtained by supposing that the mutual energy $2l$ remains at rest when the electron moves, which is evidently only

approximately true, for when the velocity is small the energy $2l$ is symmetrically distributed about the two charges e and E so that its center of mass is halfway between them and so has a velocity equal to one-half that of the electron.

The distribution of the mutual energy $2l$ about the charges can be easily calculated when their velocities are small, since it is then approximately the same as when they are at rest.

Let A be the electron and B the nucleus. At any point P the electrostatic energy density, if $AP=s$ and $BP=t$, is

$$\frac{1}{8\pi}\left(\frac{e^2}{s^4}+\frac{E^2}{t^4}+\frac{2eE\cos\theta}{s^2t^2}\right),$$

where θ denotes the angle APB.

Let $\overset{\wedge}{PAB}=\alpha$ and $\overset{\wedge}{PBA}=\beta$ and consider the mutual energy in the ring-shaped element of volume between α and $\alpha+d\alpha$ and β and $\beta+d\beta$ formed by supposing the lines bounding $d\alpha$ and $d\beta$ to revolve round AB as axis. This element of volume is equal to

$$\frac{2\pi s^2t^2\,d\alpha\,d\beta}{r}$$

so that the mutual energy in it is

$$-\frac{eE\cos(\alpha+\beta)d\alpha d\beta}{2r}$$

since $\cos\theta=-\cos(\alpha+\beta)$.

Thus the mutual energy is equal to

$$-\frac{eE}{2r}\int\int\cos(\alpha+\beta)d\alpha d\beta$$

which gives eE/r for the total mutual energy, as it should.

By means of this integral it is easy to calculate the mutual energy in any part of the space surrounding the electron and the nucleus.

The mutual energy outside a sphere described on r or AB as diameter is $1.2854\,Ee/r$ and that inside this sphere is $-0.2854\,Ee/r$. The mutual energy inside any sphere described with center at either A or B and radius less than AB is zero. Hence the mutual

energy between a sphere with center A and radius AB and one described on AB as diameter is equal to $+0.2854\ Ee/r$.

Thus it is clear that an appreciable fraction of the mutual energy is near to the electron and so might be expected to move with it and modify its motion.

To obtain information about the effect of the motion of the mutual energy with the electron, let us suppose that a fraction f of one-half of it can be regarded as moving with the electron and the rest as remaining at rest with the nucleus. Thus if $f=1$ half the mutual energy moves with the electron and half remains at rest.

The energy equation then becomes

$$\mathfrak{E}=m-\frac{m+fl}{\sqrt{1-\beta^2}}-(2-f)l. \tag{1}$$

If p denotes the angular momentum of the electron about the nucleus, we have then

$$p=\frac{m+fl}{c^2\sqrt{1-\beta^2}}r^2\dot{\theta} \tag{2}$$

where θ now denotes the angle between the radius vector r and a fixed radius in the plane of the orbit.

On working out the equation of the orbit from (1) and (2) in the usual way[1] we get writing u for r^{-1}

$$\left(\frac{du}{d\theta}\right)^2+u^2=\frac{-(2m-\mathfrak{E}-2l+2fl)(\mathfrak{E}+2l)}{p^2c^2} \tag{3}$$

$$\frac{d^2u}{d\theta^2}+u=-\frac{Ee}{p^2c^2}\left(m-\mathfrak{E}+f\frac{\mathfrak{E}}{2}-Eeu(1-f)\right) \tag{4}$$

and

$$u=C(1+\epsilon\cos\gamma\theta) \tag{5}$$

where ϵ denotes the eccentricity,

$$\gamma^2=1-\frac{E^2e^2}{p^2c^2}(1-f)$$

and

$$C=-\frac{Ee}{p^2c^2\gamma^2}\left\{m-\mathfrak{E}\left(1-\frac{f}{2}\right)\right\}.$$

Let $p_0=-Ee/C$ so that $\gamma^2=1-p_0^2(1-f)/p^2$.

[1] See for example Silberstein's *Report on the Quantum Theory of Spectra*.

Now putting the value of u given by (5) in (3) we get

$$\mathfrak{E}=m\left\{\frac{1-\alpha+\frac{\alpha f}{2}-\sqrt{1-\alpha+\alpha f}}{1-\alpha+\alpha f-\frac{\alpha f^2}{4}}\right\}$$

where

$$\alpha=\frac{p_0^2(1-\epsilon^2).}{p_0^2(1-f)-p^2}.$$

We now apply the fundamental equation of the quantum theory $\int p da = nh$ and obtain

$$p=\frac{n_1 h}{2\pi}$$

and

$$n_2 h=-p\int_0^{2\pi/\gamma}\frac{d}{d\theta}\left(\frac{1}{r}\right)\frac{dr}{d\theta}\,d\theta$$

which gives

$$n_2 h=2\pi p\gamma\left(\frac{1}{\sqrt{1-\epsilon^2}}-1\right).$$

Hence

$$\alpha=\frac{-4\pi^2 p_0^2/h^2}{\left(n_2+\sqrt{n_1^2-4\pi^2 p_0^2(1-f)/h^2}\right)^2}.$$

Since α is small we may expand $\mathfrak{E}$ in a series of powers of α, so that, neglecting powers of α above α^2, we get

$$\mathfrak{E}=m\left\{-\frac{\alpha}{2}-\alpha^2\left(\frac{3}{8}-\frac{f}{4}\right)\right\}$$

which gives on substituting the value of α and putting $k=4\pi^2 p_0^2/h^2$ and $n=n_1+n_2$

$$\mathfrak{E}=\frac{mk}{2}\left\{\frac{1}{n^2}+\frac{k}{n^4}\left(\frac{n_2}{n_1}(1-f)+\tfrac{1}{4}-\frac{f}{2}\right)\right\}.$$

Hence the frequencies of the spectral lines emitted will be given by

$$h\nu=\frac{mk}{2}\left\{\frac{1}{n^2}-\frac{1}{n'^2}+\frac{k}{n^4}\left(\frac{n_2}{n_1}(1-f)+\tfrac{1}{4}-\frac{f}{2}\right)-\frac{k}{n'^4}\left(\frac{n'_2}{n'_1}(1-f)+\tfrac{1}{4}-\frac{f}{2}\right)\right\}$$

where

$$n'=n'_1+n'_2.$$

If $f=0$, this agrees with Sommerfeld's theory as it should, and if $f=1$, the frequency ν depends on n and n' only, so that we get single lines without any fine structure at all.

For the Balmer series $n=2$ and $n'=3, 4, 5$, etc. Each line is a doublet the separation of which is got by putting $n_2=0$, $n_1=2$ and $n_1=1$, $n_2=1$ so that the separation is

$$\frac{mk^2(1-f)}{32h}.$$

If $f=0$, this is equal to 0.365 (cm^{-1}). The separation of the doublets has been measured by a number of physicists[1] and in the case of H_α the mean of ten determinations is about 0.30 (cm^{-1}). This gives

$$1-f=\frac{0.30}{0.365}$$

so that

$$f=0.18.$$

For H_β the mean of five determinations of the separation is also 0.30 (cm^{-1}); for H_γ there are only two results, 0.31, and 0.33, and for H_δ also two, 0.26 and 0.29. Thus these results seem to indicate that f may be about 0.2 for the orbits corresponding to $n=2$ of the hydrogen atom.

In the case of ionized helium the agreement between Sommerfeld's calculations of the fine structure and Paschen's observations indicates that f must be quite small.

RICE INSTITUTE
HOUSTON, TEXAS
May 10, 1922

[1] For a list of all observations see J. C. McLennan, *Proc. Roy. Soc.*, A, **100**, 217, 1921.

INTERFEROMETER OBSERVATIONS OF DOUBLE STARS[1]

By PAUL W. MERRILL

ABSTRACT

Interferometer observations of double stars.—(1) *Revised elements of the orbit of Capella* were obtained by combining the previously reported measurements of Anderson, slightly corrected, with observations made on ten nights from October, 1920, to April, 1921, using the rotating interferometer attached to the 100-inch telescope. The chief results are: $P=104.022$ days; $a=0''.05360$; $a_1+a_2=126.63\times10^6$ km; $\pi=0''.0632$; $m_1=4.2\odot$; $m_2=3.3\odot$. The average residuals are 1° for position angle and $0''.0007$ for separation. (2) κ *Ursae Majoris* was independently discovered as a double and was measured on five nights, March 1 to April 30, 1921. The separation decreased from $0''.0836$ to $0''.0772$, but the position angle changed only 2°. One component is brighter by perhaps half a magnitude. (3) ν^2 *Boötis* was found to be difficult to measure as the fringes behaved abnormally, so only a rough measurement was possible. The system may be complex. (4) *Search for new measurable double stars.* Some spectroscopic binaries, stars with composite spectra, variable stars, and some bright stars taken at random—in all 85 stars, of varying magnitude down to 5.7 (Table IV)—were examined for duplicity, but with the exception of six doubtful cases, δ Cancri, 10 Leonis Minoris, o Leonis, ϵ and f Ursae Majoris, and υ Sagittarii, no changes in fringe visibility with position angle of the interferometer were observed.

Tests by Michelson at the Yerkes and Mount Wilson observatories[2] showed that even in poor seeing interference fringes can be observed in star images with slit separations ranging from 40 to nearly 100 inches. Following these results in an attempt to make the interference method sufficiently accurate and convenient to justify its extended use in the measurement of double-stars, Mr. Anderson devised a new type of stellar interferometer with which settings are made by rotating a pair of fixed slits. This instrument he applied with great success to the determination of the relative positions of the components of Capella.[3] His measures of distance, on five nights, ranged from $0''.0418$ to $0''.0505$, while the position angles were shown to be decreasing at a rate in keeping with a revolution period of 104 days. A double-star orbit was computed by him, using the spectroscopic elements previously determined at the Lick Observatory.

[1] *Contributions from the Mount Wilson Observatory*, No. 240.

[2] *Mt. Wilson Contr.*, No. 184; *Astrophysical Journal*, **51**, 257, 1920.

[3] *Mt. Wilson Contr.*, No. 185; *Astrophysical Journal*, **51**, 263, 1920.

It is interesting to note that in 1900 J. Miller Barr called attention to the desirability of attempting the observation of Capella as a double star by interferometer methods.[1]

The present paper is a record of further applications of the rotating type of interferometer to known double stars, and to a search for new doubles of small angular separation. The 100-inch Hooker telescope was used in all the observations, and practically the same procedure as that described by Anderson was followed throughout.

The constants of the interferometer were re-determined independently. The factor which is used to reduce the actual separation of the slits to their corresponding separation in the Gauss plane was found by dividing the equivalent focal length of the telescope, 1606 inches, by 28.00 inches, the distance of the slits from the focus. The value thus obtained, 57.34, was checked by a direct determination. At the upper end of the telescope tube, on the struts supporting the secondary mirror, were placed two rectangular pieces of wood of the same shape as the slits, but with dimensions slightly smaller than those of the slits as projected in the Gauss plane. Thus, when the telescope was directed to a bright star, by removing the eye-piece and placing the eye at the focus, the blocks could be centered in the slits, leaving lines of light at the edges. The separation of the blocks, divided by the separation of the slits, gives the factor desired. This experiment yielded the value 57.2, which was thought to be a trifle small. The factor adopted for all the reductions was 57.3, which falls within the limits of uncertainty of both determinations.

CAPELLA

R.A. 1900 = 5^h9^m3; Dec. 1900 = $+45°54'$; Mag. 0.2; Class Go.

Additional interference measures of Capella have been made by the writer on ten nights, and a revised orbit has been calculated.

As the series of observations was made during the winter months poor seeing was frequently experienced. Although the observing routine could usually be carried through in the regular way, on a few occasions the conditions became so bad that reliable settings

[1] *Astrophysical Journal*, 11, 248, 1900.

could be made only by watching patiently and utilizing the best moments. The nights on which bad seeing prevents interferometer observations of Capella are quite rare. This is not true to the same extent of fainter stars.

The new measures of Capella appear in Table I, in which the third column, with the heading Slits, gives the separation (in the Gauss plane) of the slits; the fourth column, η, contains the

TABLE I

OBSERVATIONS OF CAPELLA

Date	P.S.T.	Slits	η	θ	ρ	
1920 Oct. 5....	16^h15^m	174.2 cm	46°.65	253°.9	474	
Nov. 3....	13 20	142.8	24.75	164.8	437	
	14 05	173.3	41.90	164.6	440	
	14 39	203.6	50.80	163.8	441	
1921 Jan. 14....	7 36	142.2	30.12	263.5	461	
	8 31	172.6	45.25	262.9	467	
	9 08	202.9	52.65	262.2	461	
	10 25	202.9	51.78	261.2	452	Weight ½
Jan. 31....	7 07	142.2	43.45	213.7	550	
	7 19	172.6	52.40	214.0	539	
	7 29	202.9	58.82	214.8	540	
	7 46	123.9	33.32	212.8	548	
Feb. 2....	7 18	172.6	52.67	209.8	542	
	7 52	142.2	43.30	208.6	548	
	8 55	202.9	58.07	207.9	529	Weight ½
Mar. 1....	6 52	172.6	36.95	106.0	411	
	7 10	202.9	47.00	105.6	410	
	7 23	232.8	53.25	106.2	407	
Mar. 2....	6 44	202.9	48.32	101.2	421	
	6 52	172.6	39.22	102.4	424	
	7 04	232.8	54.35	101.4	418	
Mar. 3....	6 52	172.6	39.27	98.1	425	
	7 08	202.9	48.10	97.8	419	
	7 23	232.8	54.05	98.4	415	
Mar. 31....	7 07	172.6	51.00	15.0	522	
	7 23	202.9	56.60	14.2	508	
	7 38	142.2	38.87	14.3	512	
Apr. 1....	7 16	142.2	37.50	11.4	503	
	7 35	172.6	48.65	11.4	498	
	7 55	202.9	56.32	11.7	504	

angle of projection for minimum fringe visibility;[1] the fifth column, θ, the position angle of the components; and the sixth column, ρ, their separation in units of one ten-thousandth of a second of arc, i.e., $474 = 0''.0474$. The effective wave-length was taken to be 5500 A.

In Table II are collected the mean values for each night. The weights assigned depend principally on the number of complete individual observations. The total number of settings was increased during poor seeing in an attempt to give approximately standard accuracy to each complete observation. Since the factor 57.3 is somewhat smaller than that used by Anderson in the first observations, it was decided by mutual agreement between him and the writer that it would be better to change the original reductions. Hence the first five values of ρ in Table II are larger by 2.4 per cent than the corresponding figures in *Contribution* No. 185.

TABLE II

MEAN CO-ORDINATES OF COMPONENTS OF CAPELLA

	G.M.T.	θ	O−C	ρ	O−C	Weight	
(1)	1919 Dec. 30.65.....	.....	.....	428	+ 3	½	Anderson
(2)	1920 Feb. 13.63.....	5.0	+2.2	469	− 7	⅔	Anderson
(3)	Feb. 14.65.....	1.0	+1.6	462	− 6	1	Anderson
(4)	Feb. 15.66.....	356.4	+0.5	454	− 6	1	Anderson
(5)	Mar. 15.63.....	242.0	+1.9	517	+10	⅔	Anderson
(6)	Apr. 23.63.....	107.0	+0.4	.....	.....	½	Anderson
(7)	Oct. 6.01.....	253.9	+2.6	474	− 7	½	.....
(8)	Nov. 3.92.....	164.4	−1.6	439	− 4	1	.....
(9)	1921 Jan. 14.70.....	262.6	−0.1	461	+ 5	1	.....
(10)	Jan. 31.64.....	213.8	+0.6	544	+11	1	.....
(11)	Feb. 2.66.....	208.9	+1.1	542	+14	1	.....
(12)	Mar. 1.63.....	105.9	−0.9	409	−11	1	.....
(13)	Mar. 2.62.....	101.6	−1.1	421	− 5	1	.....
(14)	Mar. 3.63.....	98.1	−0.4	420	−12	1	.....
(15)	Mar. 31.64.....	14.5	−1.0	514	+ 7	1	.....
(16)	Apr. 1.65.....	11.5	−1.0	502	+ 2	1	.....

The orbital elements deduced by Anderson gave a fair representation of the additional observations but it was found possible to make some slight improvements.

[1] The out-of-step appearance of the fringes shown in Anderson's laboratory experiments (see illustration in *Mt. Wilson Contr.*, No. 185) was not observable. Accordingly the sensitive method of fringe bisection could not be used, but settings were made for the minima of visibility.

By comparing recent spectroscopic observations at Mount Wilson with those made at the Lick Observatory in 1900 Sanford[1] has found that the original figures for the period, 104.022 days, are very nearly correct. Accordingly this value has been used throughout the present computations.

Preliminary trials showed that the eccentricity is probably smaller than 0.016 as given by Reese. Hence it did not seem necessary to vary both T and ω. The date 2422596.79 was chosen for T by trial and retained thereafter. This corresponds to a periastron time 0.8 days later than that adopted by Anderson. Corrections to a and to Ω were then found by direct algebraic processes. Finally a least-squares solution was carried through for the elements ω, i, and e. This yielded very small corrections to the preliminary values.

The adopted elements follow:

T = J.D. 2422596.79
P = 104.022 days, angles decreasing.
$\omega = 114^\circ.30$
$i = -41^\circ.08$
$\Omega = 38^\circ.70$
$e = 0.0086$
$a = 0''.05360$

To these may be added the following quantities found by combining the foregoing elements with the spectroscopic results:

$a_1 + a_2 = 126{,}630{,}000$ km $\quad m_1 = 4.2\odot$
$\pi = 0''.0632 \quad m_2 = 3.3\odot$

The residuals from these elements are given in Table II. They are considerably larger than one might expect from the degree of accordance of the individual observations as shown by Table I. Moreover, observations made on successive nights exhibit a tendency toward similar residuals. This would seem to mean either that there are systematic errors in the observations, or that the best orbital elements have not been found, or that the orbit is not a true ellipse, or that there is present a combination of two or three of these causes.

[1] *Publications Astronomical Society of the Pacific*, **34**, 178, 1922.

As to systematic errors it must be admitted there are possibilities of slight effects. For instance, no ready direct method was found of checking the position angle circle of the interferometer when attached to the telescope, so that the actual position angles of the slits may conceivably have differed by small amounts from the angles as read from the circle. Again, the condition of the telescope mirrors, the position of the struts supporting the secondary mirror, the settings of the compensator, the exact location of the focus, the condition and position of the observer's eyes, may have been responsible for small systematic errors. For the most refined results in an extensive investigation all these factors should receive attention.

In regard to the orbital elements it may be that a different set of values would give smaller residuals. Indeed it has been found by taking $T = 2422563.76$, $\omega = 0°.0$, $e = 0.01$, that the residuals in ρ are considerably reduced while the θ residuals are somewhat increased. With a circular orbit the representation is also very good. This indicates that as far as the interferometer observations are concerned the eccentricity and the time of periastron are practically indeterminate. As compared with the spectroscopic orbit this condition doubtless arises from the flexibility of the double star orbit due to the inclusion of the element representing the inclination of the orbit plane. By varying the orbital elements *arbitrarily* a better set of residuals than those in Table II could probably be found, but in view of the spectroscopic data there would apparently be no physical justification for such procedure and it has been thought better to retain the elements given on page 44 until future observations shed more light upon the situation.

One curious feature appears with several different trial sets of orbital elements, namely, that the early observations persist in giving positive residuals for θ (O−C). This may be due either to a systematic difference between Anderson's observations and the writer's, or to perturbations causing the position angle of the real node to decrease with time at the rate of about $2°.4$ per year, or about $0°.6$ in one orbital revolution. The physical interpretation and probability of such a perturbation may be a matter of interest to mathematical astronomers.

If, from the orbit adopted in this paper, we compute the position angles corresponding to the Greenwich observations[1] in 1900 (mean places), we find a fairly consistent set of residuals averaging +18°. Compared with the interferometer observations, this residual would be removed by a motion of the node of 0°.9 per year in the same direction as that suggested by the residuals of the interference measures themselves.

Capella presents one of the very small number of cases in which a comparison of the visual and spectroscopic data leads to an accurate value of the parallax. The value found in the present investigation, 0".063, agrees well with the mean trigonometric determination, 0".067, and the spectroscopic value,[2] 0".076.

κ URSAE MAJORIS

R.A. 1900 = $8^h56^m.8$, Dec. 1900 = +47°33′; Mag. 3.7; Class Ao.

On the night of March 1, 1921, Mr. Anderson and the writer were engaged in making some tests in connection with the stellar interferometer attached to the Hooker telescope, of a special apparatus[3] devised by the former to produce an artificial double star of known separation by means of which the constants used in the regular reductions might be checked. It consisted of a quartz plate two and a half millimeters thick, cut at 45° to the optic axis, which, when mounted in the converging beam, transformed a single star into a double star having a separation of 12 microns, which corresponds to 0".06 at the Cassegrain focus of the Hooker telescope. When the telescope was pointed toward κ Ursae Majoris and the observations were begun, anomalous results were obtained. Instead of the usual *four* minima in a complete revolution, a larger number appeared. We recognized that the behavior corresponded to that of a quadruple star and surmised that the star would be measurable as a double without the quartz plate. Proceeding at once to verify this by removing the quartz plate and making settings in the ordinary manner, we determined the position angle and separation to be 251°.9 and 0".0836. It was not

[1] *Monthly Notices, Royal Astronomical Society*, **61**, 72, 1901.

[2] *Mt. Wilson Contr.*, No. 199; *Astrophysical Journal*, **53**, 34, 1921.

[3] *Mt. Wilson Contr.*, No. 222; *Astrophysical Journal*, **55**, 48, 1922.

until a few days later that we learned that this star is a known double, A 1585, discovered by Aitken[1] in 1907 when its separation was about twice as great. This incident illustrates the fact that a close quadruple star can readily be detected by the interference method.

Subsequent interferometer measures made during March and April 1921 showed that the separation was decreasing at the rate of 0".003 per month, and the position angle by about one degree per month.

TABLE III

MEASURES OF κ URSAE MAJORIS

Date	Position Angle	Distance	
1907.83	283°.2	0".21	Aitken, micrometer
1909.33	281.4	0.25	Aitken, micrometer
1919.29	264.9	0.15	Aitken, micrometer
1921 Mar. 1	251.9	0.0836	.....
Mar. 2	252.5	0.0833	.....
Mar. 31	251.3	0.0801	.....
Apr. 1	250.7	0.0796	.....
Apr. 30	250.2	0.0772	.....

In good seeing the fringes are quite distinct even at minimum visibility, showing that the components are not equal. The magnitude difference is perhaps two or three times as great as in Capella, but probably does not much exceed one-half of a magnitude.

ν^2 BOÖTIS

R.A. 1900 = $15^h28^m.2$; Dec. 1900 = +41°14′; Mag. 5.0; Class A2.

At the suggestion of Dr. Aitken the double star ν^2 Boötis, A 1634,[2] was placed upon the observing list. This close double of separation 0".1 or less had been found to be a difficult object to resolve with the Lick 36-inch refractor. His observations[3] indicate that the position angle has decreased from 237° in 1907 to about 213° in 1920.

Examination of this star with the interferometer showed that the visibility changed as the slits were rotated, but observations on

[1] *Lick Observatory Bulletin*, **4**, 168, 1908.

[2] *Ibid.*

[3] Kindly communicated by letter.

different nights, and with different slit separations, did not yield the consistent results obtained with Capella and κ Ursae Majoris, and to be expected of any double star. The observations were rather difficult on account of the faintness of the image. The regular sized slits were employed although it might have been an advantage to use larger ones.

On the night of June 18, 1921, immediately following rather unsuccessful attempts at measurement, this note was written:

> Seeing and visibility are good but settings on ν^2 Boötis seem almost impossible. Fringes are certainly visible in all position angles at 108.9, 142.2, 172.6, and 202.9 cm (slit separation). It is a question of *small* differences in visibility; these differences seem smaller tonight than previously. One must guard against the physiological factor that fringes appear more distinct when parallel to the line joining the eyes. This may cause some of the above readings to be illusory. However, I believe there are real variations in visibility. Perhaps the star is triple or complex. Sometimes there appear to be regions 20° to 30° wide of low visibility. There appear to be "shoulders" in the visibility curve; i.e., the visibility drops off on rotating 5°–10° but does not return at once on rotating further. Star is a puzzle so far. Perhaps a wide double with one star a close pair.

Mr. Humason also examined this star on the same evening, June 18, 1921, and thought the variations in visibility to be real.

If we assume the object to be a double star the most probable result deduced from all the settings appears to be $\theta = 192°$, $\rho = 0''.061$. This depends very largely on observations with a slit separation of 142.2 cm. The mean epoch was May 1921.

EXAMINATION OF STARS FOR DUPLICITY

A number of stars have been tested for evidences of duplicity by looking for variations in the visibility of the fringes as the slits were rotated. Slit separations up to 200 cm (80 inches) were used in nearly all cases. The stars in Table IV exhibited no appreciable variation of fringe visibility with position angle of the slits. If these stars are double, their components could not have been as much as $0''.03$ apart on the date of observation, provided their magnitudes are nearly equal. If the difference in magnitude is greater than 0.5, changes in visibility might escape detection unless the circumstances were especially favorable. The stars

examined include spectroscopic binaries, stars with composite spectra, variable stars, and some bright stars taken at random.

TABLE IV

STARS APPARENTLY SINGLE

Star	Mag.	R.A. 1900	Star	Mag.	R.A. 1900
β Persei	var.	3^h $1^m.7$	d Boötis	4.8	14^h $5^m.8$
ζ Tauri	3.0	5 31.7	α Boötis	0.2	14 11.1
51 Aurigae	5.7	6 31.7	A Boötis	4.8	14 13.8
ψ^5 Aurigae	5.3	6 39.5	θ Boötis	4.1	14 21.8
α Geminorum	2.0, 2.8	7 28.2	ρ Boötis	3.8	14 27.5
β Geminorum	1.2	7 39.2	ϵ Boötis	2.7, 5.1	14 40.6
40 Lyncis	3.3	9 15.0	109 Virginis	3.8	14 41.2
θ Urs. Maj.	3.3	9 26.2	β Boötis	3.6	14 58.2
ϵ Leonis	3.1	9 40.2	δ Boötis	3.5	15 11.5
η Leonis	3.6	10 1.9	ι Draconis	3.5	15 22.7
α Leonis	1.3	10 3.0	β Cor. Bor.	3.7	15 23.7
λ Urs. Maj.	3.5	10 11.1	ν Boötis	5.2	15 27.3
ζ Leonis	3.6	10 11.1	α Cor. Bor.	2.3	15 30.5
μ Urs. Maj.	3.2	10 16.4	κ Serpentis	4.3	15 44.2
ρ Leonis	3.8	10 27.5	α Scorpii	1.2	16 23.3
46 Leo. Min.	3.9	10 47.7	β Herculis	2.8	16 25.9
ψ Urs. Maj.	3.2	11 4.0	α Herculis	3.5, 5.4	17 10.1
δ Leonis	2.6	11 8.8	δ Herculis	3.2	17 10.9
ξ Urs. Maj.	4.4, 4.9	11 12.9	π Herculis	3.4	17 11.6
ν Urs. Maj.	3.7	11 13.1	o Herculis	3.8	18 3.6
χ Urs. Maj.	3.8	11 40.8	d Serpentis	5.3	18 22.1
93 Leonis	4.5	11 42.8	α Lyrae	0.1	18 33.6
β Leonis	2.2	11 44.0	β Lyrae	Var.	18 46.4
β Virginis	3.8	11 45.5	113 Herculis	4.6	18 50.5
γ Urs. Maj.	2.5	11 48.6	β Cygni	3.2, 5.4	19 26.7
δ Urs. Maj.	3.4	12 10.5	δ Sagittae	3.8	19 42.9
γ Corvi	2.8	12 10.7	η Aquilae	Var.	19 47.4
η Virginis	4.0	12 14.8	31 Cygni	4.0	20 10.5
δ Corvi	3.1	12 24.7	32 Cygni	4.2	20 12.3
8 Can. Ven.	4.3	12 29.0	β Capricorni	3.2	20 15.4
δ Virginis	3.7	12 50.6	47 Cygni	4.8	20 30.0
43 Com. Ber.	4.3	13 7.2	α Equulei	4.1	21 10.8
20 Can. Ven.	4.7	13 13.1	14 Pegasi	5.0	21 45.4
ζ Urs. Maj.	2.4, 4.0	13 19.9	δ Cephei	Var.	22 25.4
17 Can. Ven.	5.0	13 30.3	η Pegasi	3.1	22 38.3
τ Boötis	4.5	13 42.5	o Androm.	3.6	22 57.3
η Boötis	2.8	13 49.9			

NOTES

$7^h 28^m.2$, α Geminorum, both components, 2.0, 2.8.
11 12.9, ξ Urs. Maj., both components, 4.4, 4.9.
13 19.9, ζ Urs. Maj., both components, 2.4, 4.0.
14 40.6, ϵ Boötis, both components, 2.7, 5.1.
17 10.1, α Herculis, both components, 3.5, 5.4.
18 22.1, d Serpentis, brighter component, 5.3.
19 26.9, β Cygni, both components, 3.2, 5.4.

Although the dates of observation, slit separations employed, the seeing, and notes on any apparent peculiarities were, of course, recorded in the observing book and will be made available to anyone interested in a particular star, these details do not seem worth publishing. For the most part the dates of observation lie between February 1 and June 17, 1921. Several stars observed by Anderson about a year earlier are included in Table IV. On a few nights Mr. Humason alternated with the writer in the examination of stars for duplicity.

Variations in visibility for f Ursae Majoris (R.A. 1900 = 9^h1^m9; Dec. 1900 = $+52°0'$; Mag. 4.5; Class A3p) were suspected on the night of April 30, 1921, but a complete set of minima as for a double star was not obtained and the variations were not definitely confirmed.

The following stars were also suspected of changes in visibility, but the reality of the changes is doubtful. They do, however, deserve careful re-examination, which it is hoped other observers will give them, as the double star work at the Mount Wilson Observatory has been discontinued.

	Mag.	R.A. 1900
δ Cancri	4.2	8^h39^m0
10 Leo. Minor	4.6	9 28.1
o Leonis	3.8	9 35.8
ϵ Urs. Maj.	1.7	12 49.6
ν Sagittarii	4.6	19 16.0

In searching for doubles with the interferometer, poor and, in particular, variable seeing is a great disadvantage and renders conclusions uncertain. To illustrate this the following quotations from the observing record may be of interest.

γ Boötis R.A. 1900 = 14^h28^m1; Dec. 1900 = $+38°45'$; Mag. 3.0; Class F.

1921 April 1 Seeing fair. "Several times suspected slight variations but cannot make any settings. Probably single."

June 16 Seeing poor. "Visibility very low on account of poor seeing. Fringes seen only by rare glimpses."

June 18 Seeing fair. "Round."

γ Lyrae — R.A. 1900 = $18^h55^m.2$; Dec. 1900 = $+32°33'$; Mag. 3.3; Class A.

1921 April 30 — Seeing good but rather irregular. "Suspect slight variation but may not be real."

June 14 — Seeing poor and variable. "Followed around at 142 cm with some suspicion as to variability (doubtless due to variable seeing). Followed partly around at 173 cm but seeing turned very poor and fringes disappeared."

June 17 — Seeing poor. "About equally distinct in all position angles at 142 cm and 197 cm. Fairly well seen by waiting for best moments."

While complete disappearance of the fringes, as for a close double of equal components, would be noticed in any except the very worst seeing, small variations in visibility cannot be recognized with certainty in poor and irregular seeing. Thus the suspected doubles noted in preceding paragraphs may be spurious, while some of the stars in Table IV may prove to have small variations in visibility, due to unequal components, which were overlooked by the writer. The brighter the star the less the difficulty caused by the seeing.

GENERAL REMARKS

In the Mount Wilson interferometer the slits are separated by turning a divided head, the readings of which are calibrated in terms of the actual slit separation. For systematic double star observations it would probably be better to replace this design by a metal plate containing pairs of slits at several fixed separations so arranged that any pair could be easily moved to the center of the field, the others remaining covered. Time would be saved in observing and the possibility of errors in setting and reading the separations greatly reduced. The separations, moreover, would not be subject to change with time or with readjustment of the instrument, unless the distance of the plate from the focus were changed.

The accuracy of the results obtained by Anderson's method of rotating the apertures and the convenience with which it can be applied are so great as to make it probable that this method will become very important in the future of double star astronomy. The range in the separations and magnitude differences of the double stars to which it can be successfully applied is, of course,

small, but the interferometer results would supplement the ordinary micrometer measures in a most desirable way, making it possible to follow some very close and especially interesting pairs with highly satisfactory precision. The new pairs discovered by the interferometer will not be numerous as compared to those now known through direct observations, but they are likely to be of very great individual importance.

The present investigation has been greatly facilitated by Mr. Anderson's kindness in placing his experience with the stellar interferometer in so far as possible at the disposal of the writer, and by considerable assistance in securing and recording the observations. This kindness is gratefully acknowledged, as well as that of Mr. Humason in aiding with the observations on several nights.

MOUNT WILSON OBSERVATORY
April 1922

A DETERMINATION OF THE WAVE-LENGTHS OF 118 TITANIUM LINES BY MEANS OF THE INTERFEROMETER

By FREDERICK L. BROWN

ABSTRACT

Interferometer measurements of the wave-lengths of 118 titanium lines from a vacuum arc, λλ 4263 to 6261 A.—Because of the importance of the titanium spectrum in astrophysics, a more accurate determination of the wave-lengths was undertaken, using a Fabry-Perot interferometer in connection with a Hilger constant deviation spectrograph. The sources of light were two water-cooled vacuum arcs, one for the titanium and the other for the cadmium lines. In order that both spectra might be photographed simultaneously an image of the Ti arc was formed, by means of a concave mirror, at the center of the Cd arc, thus making the paths through the optical system the same for both. Two etalon separators, 9.81 and 4.99 mm thick, were used to enable the phase correction to be determined from a comparison of the uncorrected wave-lengths obtained with the two separators, five plates being made with each. In reducing the measurements, Kilby's wave-lengths were used as a point of departure and the Cd line 6438.4696 as standard. Temperature and pressure-corrections were negligible. The results from the various plates agree on the average within 0.002 or 0.003 A.

Interferometer measurements of the wave-lengths of 25 iron lines and two manganese lines from a vacuum arc are also included. Making allowance for the pressure-shift as given by Gale and Adams, the mean deviation from the results of Burns, Meggers, and Merrill for the iron lines is ±0.003 A. For the Mn lines λλ 4783 and 4823, the deviation from the measurements of Goos is −0.020 A, perhaps a pressure-effect.

INTRODUCTION

The measurement of some of the principal lines in the spectrum of titanium with the interferometer was undertaken in order to supplement the data already available on this important element.

The first measures of titanium lines were made by Thalén at Upsala in 1868.[1] Later, Hasselberg,[2] also Exner and Haschek[3] measured a large number of lines in both arc and spark. Kilby,[4] at Johns Hopkins in 1909, measured several hundred lines between λ 2800 and λ 6300 with a Rowland grating, interpolating between the iron standards. The effect of pressure-shift has been examined

[1] Thalén, *Nova Acta Reg. Soc. Sc. Upsala*, Ser. III, **6**, 1868.

[2] Hasselberg, *Astrophysical Journal*, **4**, 116–134 and 212–233, 1896.

[3] Exner and Haschek, *Die Spektren der Elemente bei Normalen Druck*, 1911.

[4] Kilby, *Astrophysical Journal*, **30**, 243–266, 1909.

by Humphreys, Haschek, Gale, and Adams;[1] induction and capacity shifts have been studied by Kent and Middlekauf;[2] the Zeeman effect and electric furnace spectra, by A. S. King;[3] E. J. Evans[4] has extended the measures of wave-lengths down to λ 7364; Kiess and Meggers[5] down to λ 9743; and various other observers have studied the several phases of the arc and spark spectra of this metal;[6] there has, however, so far as the author is aware, been no previous measurement of the spectrum of titanium with the interferometer.

The value of titanium as a comparison spectrum was first pointed out by Frost[7] who found it especially good for solar and stellar work on account of the wealth of bright sharp lines in the region λ 4200 to λ 5200. Furthermore, Jewell,[8] studying the chromosphere, has found it of service as a comparison spectrum in the violet and ultra-violet region.

THEORY

The contest between the grating and the interferometer as the fundamental method for wave-length determination was decided in favor of the latter through the masterly work of Michelson.[9] In the hands of Fabry and Perot the interferometer developed into an instrument of precision and of great experimental possibilities. Used in connection with even a moderate dispersion piece this device makes possible the accurate measurement of a large number of lines on a single photograph.

[1] Humphreys, *Astrophysical Journal*, **6**, 169–232, 1897; Haschek, *ibid.*, **14**, 181–201, 1901; Gale and Adams, *ibid.*, **35**, 10–47, 1912, and **37**, 391–394, 1913.

[2] Kent, *ibid.*, **17**, 286–299, 1903; **22**, 182–198, 1905; **27**, 70–77, 1908; Middlekauf, *ibid.*, **21**, 116–150, 1905.

[3] King, *ibid.*, **28**, 300–314, **29**, 76–83, 1908; **30**, 1–13, 1909; **31**, 433–458, 1910; **34**, 225–250, 1911, etc.

[4] Evans, *ibid.*, **29**, 160–163, 1909.

[5] Kiess and Meggers, *Sci. Papers, Bureau of Standards*, No. 372, 1920.

[6] Kayser, *Handbuch der Spectroscopie*, **6**, 655–708, 1912.

[7] Frost, *Astrophysical Journal*, **10**, 207–208, 1899.

[8] Jewell, *ibid.*, **11**, 243–244, 1900.

[9] Michelson and Benoit, *Travaux et Memoirs du Bureau Intern. des Poids et Mesures*, **11**, 1895.

The theory[1] of the instrument is so well known that it need not here be given. The laboratory equation without corrections of any kind as given by Pfund[2] is:

$$\lambda = \frac{\lambda_s P_s}{P}\left(1 + \frac{D_s^2}{8R_s^2} - \frac{D^2}{8R^2}\right)$$

where λ is the wave-length to be determined; P is the order of interference (approximately $\frac{2e}{\lambda}$, where e is the distance between the etalon plates) for the bright ring measured; D is the linear diameter of this ring; R is the effective focal length of the optical train for light of wave-length λ; and λ_s, P_s, D_s, and R_s are the corresponding quantities for a bright ring of the system formed by the standard red cadmium line.

To this computed value of λ two principal corrections must be applied, one for phase, the other to reduce the computed wave-lengths to standard conditions of pressure and temperature. The wave-length of the red cadmium ray which has been adopted as the primary standard by the International Solar Union[3] is 6438.4696 A, the etalon space being filled with dry air at 760 mm of mercury and 15° C., and the radiation produced in vacuum.

DESCRIPTION OF APPARATUS

1. *Source.*—The sources of radiation in the present experiment are two water-cooled vacuum arcs. They consist of two concentric cylindrical brass vessels so arranged as to give a fairly large chamber for the arc; the space between the walls of the vessels form a water jacket. The accompanying diagram (Fig. 1) shows a vertical section with the electrodes in place.

The heavy cone-shaped top C fits snugly against the upper ring D in such fashion as to render the joint air tight and allow the main chamber to be exhausted through a side tube.

[1] Fabry and Perot, *Astrophysical Journal*, **15**, 73–96 and 261–273, 1901.

[2] Pfund, *ibid.*, **28**, 197–211, 1908.

[3] *Transactions of the International Union for Co-operation in Solar Research*, **2**, 109–137, 1907.

The electrodes are held in place by two insulating frames, *HH*, which slide up and down a pair of guides and are operated by means of a right- and left-handed screw *JJ* so as to be brought together and then separated to any desired distance after the arc has started. The current is introduced at the terminals *KK*, which are connected to the electrodes by stranded copper wire incased in short lengths of clay pipe-stem. It is evident from the diagram

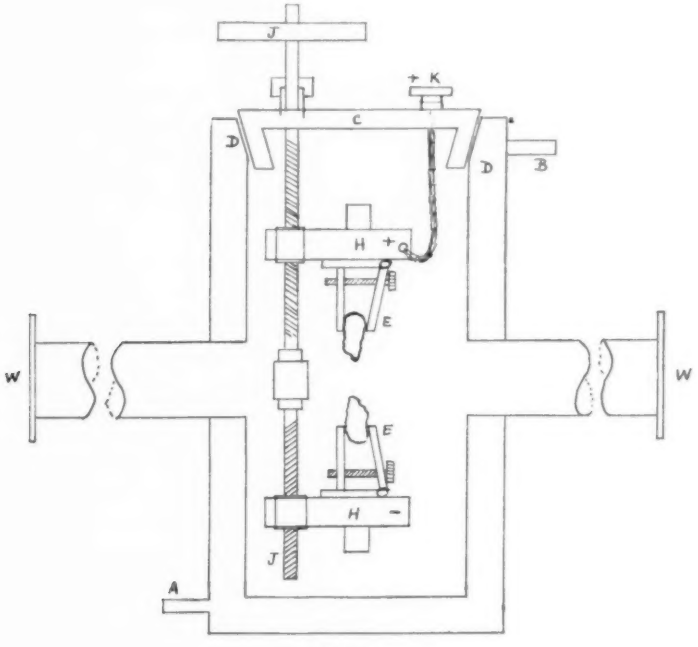


FIG. 1

that the entire system of electrodes, clamps, etc., can be removed by lifting off the top.

The water for cooling the chamber enters the outer can at the bottom through *A* and leaves at the top through *B*. The arc is adjusted so that the light from it passes directly out of the windows *WW*. The long tubes keep the glass from becoming soiled and spattered by the arc.

A vacuum of five or six millimeters of mercury could be maintained very easily. The arcs worked best with a pressure of about

10 mm for Cd and about 40 mm for Ti. Under these conditions the light is steadier and brighter than under a pressure of only one or two millimeters. They were supplied with direct current from a 220-volt generator, the titanium arc using from 5 to 7 amperes while the cadmium arc took from 0.5 to 2.0 amperes.

The electrodes for the Cd arc were carbon, the lower one being bored out and filled with metallic cadmium. The electrodes for the titanium arc were small wedge-shaped pieces of metallic titanium clamped in iron jaws. The arc was usually quite steady, but would occasionally jump to the iron clamps thereby introducing the iron impurity lines measured on some of the plates. The greatest difficulty with the cadmium arc arose from the easy volatilization of the metal and its condensation on the electrodes and their insulating supports, thus producing short circuits from time to time that required the removal and thorough cleaning of the entire apparatus. The cadmium often had to be replenished two or three times during a single exposure.

The vacuum vessels were mounted on sliding ways so that by means of adjusting screws the light could be kept accurately centered on the diaphragm of the etalon even when the arc moved from one point of the electrode to another.

I am glad to remark here that the design of this arc is due to Professor Henry Crew and Dr. George V. McCauley,[1] and that the apparatus was made with great accuracy by Mr. F. Küng, the mechanician of the department.

2. *Interferometer and optical train.*—The interferometer was made by Hilger for use with his spectrograph. The plates were cleaned and resilvered by a modified form of the process described by Wood.[2] The deposit was good over the central area, the only portion used, and gave clear sharp rings. The same portions of the mirrors were used throughout the experiment.

The original interferometer has a quartz etalon of 9.81 mm separation, in addition to which a second one, of 4.99 mm separation, was also made in order to determine the phase correction. A diagram of the optical train is given in Figure 2.

[1] Crew and McCauley, *Astrophysical Journal*, **39**, 29–38, 1914.

[2] Wood, *Physical Optics*, 2d edition, p. 281.

The light from the titanium arc, *A*, is brought to a focus at the center of the cadmium arc, *C*, by the concave mirror, *B*. From here on the two pencils, one from each source, travel the same path.

The lens, *D*, collects and focuses the rays on the diaphragm, *E*, in front of the etalon *F*. At *G*, there is placed an achromatic lens which projects the ring system on the slit-plate of the spectrograph, of which *H* is the collimator, *I*, the prism and *J*, the camera.

The special and most important feature of the arrangement is the use of the concave mirror, *B*, which enables one to use both arcs simultaneously. In this way the optical system for both Cd and Ti is identical from the crater of the Cd arc to the plate. No changes

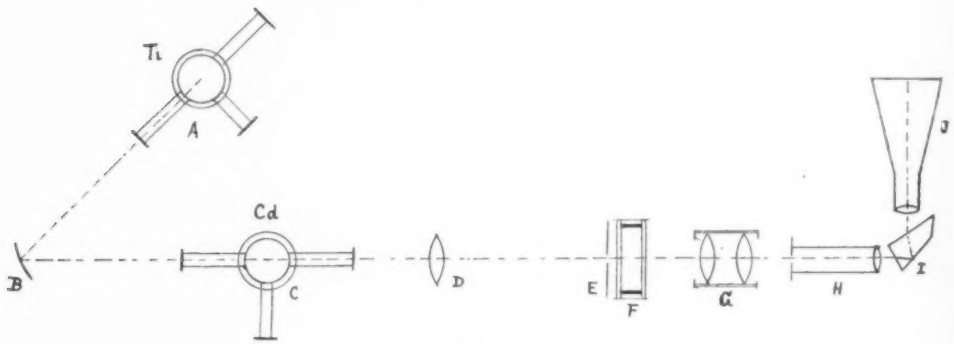


FIG. 2

being required, the chances of shift of the rings on the plate due to alternate exposures is entirely eliminated. This important suggestion was made by Professor Crew who had used a similar arrangement for work with a large grating.

3. *The spectrograph* is one of the constant-deviation prism type made by Hilger. The collimator and camera lenses are very good, and the adjustments on the camera make it possible to secure good definition over the entire range of the plates.

There are two disadvantages in the use of a prism for this work, namely, its low dispersion in the orange-red region and the absorption of the short wave-lengths. However, titanium is richest in lines shorter than 5200, and the use of glass plates in the interferometer and glass lenses in the rest of the optical train cuts out almost as much of the ultra-violet as does the prism.

METHOD OF EXPERIMENT

The adjustment of the optical train was carried out in the usual manner, care being taken to make certain that the image of the titanium arc, *A*, formed by the concave mirror, *B*, coincided with the center of the cadmium arc, *C*, and that the light from both sources followed the same path through the optical train. The diaphragm, *E*, not only delimits the portion of the mirrors used, thereby sharpening the rings, but also cuts off light from the poles of the arc and allows only that from the center to reach the plates of the etalon.

The photographic plates used were Wratten and Wainwright panchromatic, $3\frac{1}{4}\times 4\frac{1}{4}$ inches; to make them more sensitive to the red and faster over the entire range, they were bathed in ammonia and dried in an air-current shortly before exposure.[1] The camera gave a spectrum whose width was about three-fourths of an inch.

When the adjustment of the instrument and good condition of the arcs had been assured, the exposure was begun and the arcs kept running for some three hours. Stopcocks in the vacuum pump line allowed one arc to be opened to the air and its electrodes replaced without disturbing the other one. In this way the exposure was continuous, and during the major portion of the time both sources were in simultaneous operation.

On a great many plates a direct exposure, i.e., a photograph without the etalon was put on the plate, either above or below the ring system which was, of course, in the middle. These direct exposures required only a few seconds and were added after the interference fringes had been photographed. They gave a convenient comparison spectrum in case of question of identification of lines. The width of the lines on the photographic plates varied from 0.10 to 0.15 mm for the interference fringes; for the direct exposures it was of course much narrower.

The list of plates used in the final computation is given in Table I. It will be noticed that some were taken in July and August 1920, and some in February 1921. The set of plates made with the 4.99 mm etalon in the summer of 1920 looked beautiful and gave promise of good results, but something must

[1] Burka, *Journal of the Franklin Institute*, **189**, 25–46, 1920.

have been out of adjustment and escaped detection, for the wavelengths computed from them proved to be inconsistent among themselves as well as at variance with the values given by the 9.81 mm etalon. Consequently the separator of the short etalon was reground and the entire apparatus readjusted. Two addi-

TABLE I

PLATES MEASURED

Pate Number	Date	Time of Day	Source	Exposure Length	Current Amps.	Room Temperature	Atmospheric Pressure
B 63...	1920 July 30	9:00–12:00	Cd Ti	3 hrs. 3	1–2 6–7	24°.5 C.	745 mm
B 64...	July 30	2:30– 5:30	Cd Ti	3 3	1–2 6–7	24.5	744
B 75...	Aug. 7	9:00–12:00	Cd Ti	2+ 3	1–2 5–7	25.0	746
B 76...	Aug. 7	1:30– 5:00	Cd Ti	2:15 3	1–2 5–7	25.0	745
B 88...	1921 Feb. 5	3:00– 6:00	Cd Ti	1:30 2+	1 5–7	20.0	742
B 89...	Feb. 8	3:00– 5:00	Cd Ti	1+ 2–	1 5–7	20.0	745
B 90...	Feb. 10	2:00– 5:30	Cd Ti	2+ 3+	1–7 5–7	21.0	742
B 91...	Feb. 12	2:30– 5:30	Cd Ti	2+ 3–	0.5–1 5–7	22.0	745
B 93...	Feb. 17	2:30– 4:30	Cd Ti	1:30 2	0.5–1 5–7	20.5	754
B 98...	Feb. 19	2:30– 5:30	Cd Ti	1 2:30	0.5–1 5–7	20.5	757

9.81 mm etalon used for plates Nos. 63, 64, 75, 76, 98.
4.99 mm etalon used for plates Nos. 88, 89, 90, 91, 93.

tional plates with the long etalon were also made as a check and one of these (No. 98) was measured and completely reduced. The new set of plates are in good agreement and there is no question but that the difficulty, still unknown, that vitiated the first set was entirely removed.

MEASUREMENT OF THE PLATES

The plates were all measured at the Dearborn Observatory on a Gaertner engine designed for stellar parallax plates. Through the kindness of Professor Philip Fox, this instrument was made available and greatly facilitated the work, since it has two mutually perpendicular screws. These make it possible to measure all the rings on all lines without touching the plate, and since the dispersion is not large, it makes identification of the ring systems a simple matter. The machine is equipped with half-millimeter screws which enable one to read settings directly to 0.001 mm.

The plate was placed on the machine so that the spectral lines were perpendicular to the horizontal screw, and the position of each good line was located by means of this screw. This position was necessary to identify the line. Then by use of the other screw the diameters of the different rings in each line were measured.

In general, three settings were made on each ring, the mean being used in subsequent computation. For the fine sharp rings of the red cadmium and other similar lines, the settings on any one ring are almost identical, while, for very wide, hazy, or overexposed ring systems, they differ by as much as three or four hundredths of a millimeter. For the ordinary line, the two inner rings were measured except when the innermost ring was too small, or too hazy, in which case, the second and third were chosen. However, for the four cadmium lines and for ten titanium lines scattered through the spectrum, the innermost three or four rings and also the outermost three or four rings were measured. These fourteen special lines were used in the computation of R, the effective focal length of the optical train. As a further precaution and check, these special lines were generally measured twice, once before the general lines were measured and again afterward. The different values of the ring diameters gave an idea of the setting error and made it clear that for many rings it is not possible to repeat the measures closer than 0.01 mm; for sharp rings the deviation is less than half this amount. Throughout the computation all figures are retained to 0.001 mm to prevent errors from accumulating. On some of the plates a few iron impurity lines

show; on another two manganese lines were strong enough to permit measurement.

The intensity and quality of each ring system was judged and recorded at the time of measurement. The author used 0 to 20 for intensity gradings, but in the final table the means of his values have been reduced to a scale of 0 to 10 to agree with general usage. The quality was denoted as poor (p), fair(f), good (g), very good, (vg), and the two intermediate ones (fp) and (fg).

Frequently it was not possible to measure an entire plate at one sitting and since the instrument was in constant use, the plate had to be removed between times; but this introduced no error into the results, for each line has its own independent ring system, and only diameters which are the differences of settings made at the same time are used in the computation.

REDUCTION OF MEASURES

The computation was carried out in much the same manner as that described by Pfund[1] so need not be given in detail here. Kilby's wave-lengths[2] for the titanium lines were used as a point of departure. The exact order of interference was determined by the method of Lord Rayleigh,[3] using the best available wave-lengths of the four cadmium lines. Then by use of the laboratory equation given earlier in this paper, the uncorrected wave-length of each line on each plate was computed.

CORRECTIONS

1. *Phase change.*—The next question is the correction to be applied for change of phase. If one assumes, according to the general view, that change of phase is produced by waves of different length penetrating the silver film to different depths, then it is clear that one has a different etalon distance for each different line on the spectrum.

The wave-lengths computed above were of course based on the assumption that the etalon distance was the same for all, namely,

[1] Pfund, *Astrophysical Journal*, **28**, 197–211, 1908.

[2] Kilby, *ibid.*, **30**, 243–266, 1909.

[3] Rayleigh, *Philosophical Magazine*, **11**, 685–703, 1906.

its effective distance for the standard Cd ray. If, however, it has a different value for the blue ray, then the order of interference should be corrected for this difference in path, but this amounts to a correction to λ. Furthermore, since the depth of penetration of the rays is independent of the etalon distance, its relative value will be large for a short etalon and almost negligible for a very long one. For the case in hand it will be nearly twice as great for the 4.99 mm as for the 9.81 mm etalon. In fact the ratio of the two distances is 1.92+ and since the correction will be small in any case we may without appreciable error call this ratio exactly two. For the standard ray we may assume that the phase change is zero in both cases. Consequently the uncorrected values of λ given by the short etalon should be just twice as far from the correct value as those given by the long one; or what amounts to the same thing, *the difference between the uncorrected values given by the two etalons is equal to the phase correction which must be applied to the longer etalon values; and is half of what must be applied to the short etalon values.*

For if

$$\lambda=\lambda'+d$$

for the long etalon and,

$$\lambda=\lambda''+2d$$

for the short etalon, then subtracting,

$$0=\lambda''-\lambda'+d \text{ or } d=\lambda'-\lambda''$$

where λ is the correct value; λ' and λ'' are the approximate values given by the long and short etalon respectively and d is the phase correction to be applied to the long etalon values. From this it is evident that we can calculate the phase correction as soon as pairs of uncorrected wave-lengths from the two etalons are known.

The values of this correction obtained from the fourteen special lines were plotted against wave-lengths. Then for additional information the values of the correction for all other lines appearing on all ten plates were grouped by zones of wave-length and the mean correction for each zone plotted. Through this double set of points a single curve was drawn and the phase correction given by this curve was applied to the uncorrected values of the wave-lengths.

A glance at the curve will show that the points are scattered principally over the range from 0.0040 to 0.0060 A, between λ 4200 and λ 5400 and it was felt that the straight line $Y = 0.0050$ represented as well as any curve this portion of the graph. From λ 5400 to λ 6400 there is a serious lack of data that makes the shape of the curve in this region a mere guess. Certain it is that it must be zero at λ 6438 and the two values near λ 5900 give the only other information.

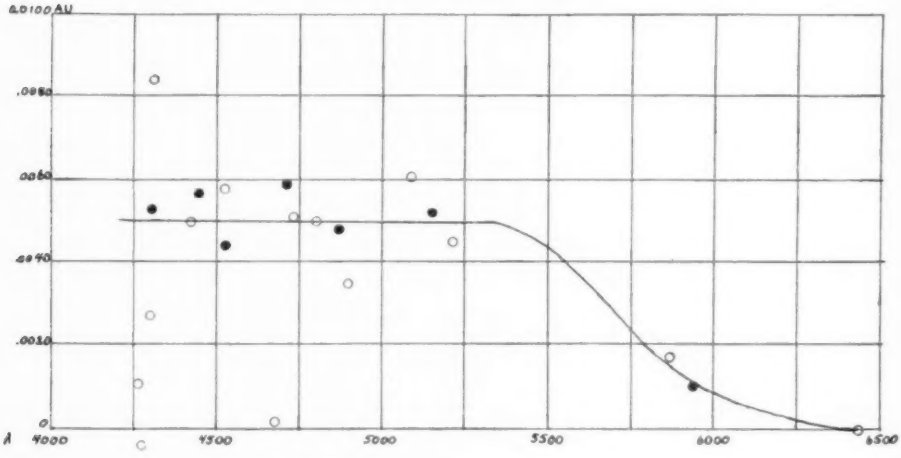


FIG. 3.—Phase change curve

In drawing the curve the values of λ 4263 and λ 4274 are left out of consideration because these lines do not appear on all the plates and are difficult to measure on many of the plates where they do show. Likewise, the values of Cd 4678 are discarded, for the line is known to be complex and it is so close to the stronger line Ti 4681 that it is very hard to measure.

2. Still another correction must be applied in order to reduce the wave-lengths to standard conditions of pressure and temperature (760 mm of mercury and 15° C.) for the air in the etalon space. The maximum value of this correction is of the order of 0.001 A and it was felt that in view of the small relative size of

this error compared to the accidental setting error, and the amount of labor required to compute it for all lines on all plates, it might be omitted without causing appreciable error in the final results.

3. In the case of the standard iron lines measured, a different kind of correction must be considered, before comparing the values in this table with the International Standards. These standards are obtained from a Pfund arc operated in air at a pressure of one atmosphere; while the source used in this work was in a vacuum. One would expect a pressure-shift. This will be spoken of later.

DISCUSSION OF RESULTS

A comparison of the computed values of the wave-length for any one line given by the short etalon plates shows them to be less in accord among themselves than the values given by the long etalon plates, as indeed one would naturally expect, since the order of interference is greater for the long etalon than for the short. Consequently the values from the short etalon have been given a weight of 2, while those from the long etalon have been given a weight of 3, in computing the mean value. The final wave-length for any line given in Table II is found by correcting each plate value for phase change and taking the weighted mean of these corrected values.

Table II gives the final values for all the lines measured and is arranged as follows: Column 1 gives the weighted mean of the plate values computed in this work. Column 2 gives the wave-length according to Kilby[1] for the titanium lines, and according to Burns, Meggers, and Merrill[2] for the iron lines, and according to Goos[3] for the manganese lines and a few iron lines not given in the Bureau publication. Column 3 gives the average deviation of the individual measures from the mean. Column 4 gives the number of plates on which the line was measured. Column 5 gives the mean value of the intensity on a basis of 10. Column 6 gives the mean value of the quality of the line. Column 7 shows the difference between the values in the first two columns taken in the sense

[1] Kilby, *Astrophysical Journal*, **30**, 243–266, 1909.

[2] Burns, Meggers, and Merrill, *Sci. Papers, Bureau of Standards*, No. 274, 1916.

[3] Goos, *Astrophysical Journal*, **35**, 221–232, 1912; **37**, 48–59, 1913.

TABLE II

	Brown Wave-Length in I.A.	Kilby Wave-Length in I.A.	Mean Deviation	No. Plates Measured	Intensity	Quality of Lines	Diff. λ Kilby − Brown	Pressure-Shift per Atm.	Notes
Fe	4250.794	.791	.....	1	1	fp	−.003	+.003	
Fe	4260.474	.489	.000	2	2	f	+.015	+.006	
	4263.134	.142	.003	5	3	fg	+.008	+.003	(1)
Fe	4271.760	.764	.003	5	3	fg	+.004	+.003	
	4274.585	.600	.002	4	3	g	+.015	.....	(1)
	4282.696	.715	.000	2	2	g	+.019	+.003	
	4286.006	.012	.002	8	3	fg	+.006	+.003	
	4287.406	.412	.004	2	2	f	+.006	+.003	
	4289.072	.080	.001	4	3	f	+.008	+.003	
	4290.932	.936	.002	8	3	fg	+.004	.....	
	4294.103	.102	.....	1	1	fp	−.001	+.004	
	4295.751	.755	.002	8	3	fg	+.004	+.003	
	4298.666	.666	.002	10	4	fg	.000	+.003	(1)
	4305.910	.907	.001	9	8	fg	−.003	+.003	
Fe	4307.899	.910	.002	5	3	fg	+.011	+.003	
	4314.800	.800	.002	10	5	g	.000	+.002	(1)
	4318.630	.635	.002	5	2	fg	+.005	.....	
	4321.652	.659	.002	2	3	g	+.007	.....	
Fe	4325.762	.770	.003	4	5	fg	+.008	+.003	
	4337.915	.915	.004	2	2	fg	.000	+.002	
Fe	4383.544	.548	.002	6	8	vg	+.004	+.003	
	4395.036	.037	.003	3	3	fg	+.001	+.003	
	4404.276	.273	.003	5	4	g	−.003	+.003	
Fe	4404.749	.752	.002	4	7	g	+.003	+.003	
Fe	4415.124	.127	.004	3	4	g	+.003	+.002	
	4417.275	.278	.003	4	4	g	+.003	+.003	
	4427.098	.102	.001	10	7	g	+.004	+.002	(1)
	4440.345	.343	.....	1	3	vg	−.002	+.004	
	4443.799	.801	.003	4	4	fg	+.002	+.003	
	4449.143	.127	.003	10	7	g	−.016	+.004	
	4450.893	.902	.005	2	3	fg	+.009	+.004	
	4453.308	.323	.003	3	5	g	+.015	+.005	
	4455.319	.324	.005	2	3	fg	+.005	+.005	
	4457.427	.427	.002	10	7	g	.000	+.005	
	4465.807	.803	.002	8	3	g	−.004	+.003	
	4468.493	.493	.001	5	4	fg	.000	+.006	
	4471.239	.235	.002	8	3	g	−.004	+.003	
	4481.259	.267	.002	7	5	fg	+.008	+.003	
	4489.089	.096	.002	9	5	g	+.007	+.004	
	4496.146	.150	.003	9	4	g	+.004	.....	
	4501.269	.271	.002	9	3	g	+.002	+.006	
	4512.734	.736	.002	10	7	g	+.002	+.004	
	4518.024	.026	.002	10	8	g	+.002	+.004	
	4522.797	.803	.002	10	7	g	+.006	+.004	(11)
	4527.305	.315	.002	10	7	g	+.010	+.004	(1)
	4544.689	.700	.002	10	7	vg	+.011	+.004	
	4548.767	.772	.002	7	9	g	+.005	+.004	
	4552.454	.400	.002	10	7	g	.....	+.004	(12)
	4555.485	.498	.002	10	7	vg	+.013	+.004	
	4563.757	.765	.003	9	3	fg	+.008	+.004	
	4571.970	.981	.002	10	6	g	+.011	+.006	
	4617.270	.268	.002	10	8	vg	−.002	+.004	

TABLE II—*Continued*

	Brown Wave-Length in I.A.	Kilby Wave-Length in I.A.	Mean Deviation	No. Plates Measured	Intensity	Quality of Lines	Diff. λ Kilby − Brown	Pressure-Shift per Atm.	Notes
	4623.098	.098	.002	10	6	vg	.000	+.003	
	4629.337	.335	.002	10	4	g	−.002	+.005	
	4645.194	.188	.001	2	4	g	−.006	+.005	
	4650.016	.017	.003	2	3	fg	+.001	+.005	
	4656.468	.474	.002	5	7	g	+.006	+.002	
	4667.585	.589	.001	5	7	g	+.004	+.003	
Cd	4678.155	.152	.003	10	6	g	−.003	.....	(1), (2)
	4681.909	.914	.003	10	8	g	+.005	+.002	(1)
	4691.334	.341	.003	10	4	g	+.007	+.005	
	4698.763	.790	.002	9	3	g	+.027	+.005	
	4710.187	.194	.002	10	3	g	+.007	+.005	
	4722.603	.621	.001	4	3	g	+.018	+.005	(3)
	4723.171	.179	.001	4	3	g	+.008	+.005	(3)
	4742.700	.801	.003	10	5	vg	+.011	+.005	
	4778.263	.271	.....	1	2	g	+.008	+.006	
Mn	4783.415	.437	.....	1	1	fg	+.022	.....	
	4792.484	.499	.002	4	2	fg	+.015	.....	
Cd	4799.915	.911	.002	10	9	g	−.004	.....	(1), (4)
	4805.418	.434	.003	9	3	fg	+.016	+.009	
	4820.412	.420	.002	10	4	vg	+.008	+.004	
Mn	4823.507	.526	.....	1	3	vg	+.019	.....	
	4840.874	.878	.002	10	7	vg	+.004	+.001	
	4856.012	.013	.003	10	6	vg	+.001	+.004	
	4868.261	.286	.003	6	3	fg	+.025	+.003	
	4870.129	.141	.003	7	5	fg	+.012	+.004	
	4885.082	.093	.002	10	7	vg	+.011	+.003	
Fe	4890.773	.771	.001	3	3	g	−.002	+.009	(5)
Fe	4891.490	.510	.001	3	3	g	+.020	+.007	(5)
	4899.911	.925	.003	10	7	vg	+.014	+.002	(1)
	4913.617	.625	.002	10	6	vg	+.008	+.003	
Fe	4920.509	.521	.....	1	.....	.....	+.012	+.001	
	4928.339	.357	.003	9	3	fg	+.018	+.004	
	4938.291	.306	.....	1	3	g	+.015	.....	
Fe	4957.594	.612	.001	5	4	g	+.018	+.011	
	4981.734	.746	.002	9	10	g	+.012	+.003	
	4991.065	.077	.002	9	9	g	+.012	+.004	
	4999.502	.512	.002	9	9	g	+.010	+.004	
	5007.211	.218	.003	9	9	g	+.007	+.003	
	5014.283	.262	.002	3	7	g	−.021	+.003	
	5016.165	.174	.005	3	3	f	+.009	+.003	
	5020.023	.045	.002	3	4	g	+.022	+.005	
	5022.864	.872	.006	3	5	f	+.008	+.004	
	5024.831	.853	.002	3	4	fg	+.022	+.003	(6)
	5025.589	.580	.002	3	4	fg	−.009	.....	(6)
	5039.953	.962	.003	5	6	fg	+.009	.....	
	5064.656	.659	.003	10	7	vg	+.003	+.002	
	5071.475	.490	.....	1	5	fg	+.015	+.010	
Cd	5085.823	.822	.002	10	8	vg	−.001	.....	(1), (7)
	5113.444	.441	.003	7	3	g	−.003	+.004	
	5120.419	.429	.003	10	3	g	+.010	+.001	
	5145.463	.470	.002	4	3	fg	+.007	+.004	
	5147.487	.476	.....	1	4	vg	−.009	+.002	

TABLE II—*Continued*

	Brown Wave-Length in I.A.	Kilby Wave-Length in I.A.	Mean Deviation	No. Plates Measured	Intensity	Quality of Lines	Diff. λ Kilby − Brown	Pressure-Shift per Atm.	Notes
	5152.185	.184	.003	4	3	fg	−.001	+.002	
Fe	5167.491	.495	.003	2	3	g	+.004	.....	
	5173.744	.744	.002	10	7	vg	.000	+.003	
	5192.979	.978	.002	10	7	vg	+.008	+.002	
	5210.386	.394	.001	10	8	vg	+.008	+.002	(1)
	5224.304	.323	.....	1	.....	.....	+.019	.....	(13)
	5224.946	.960	.....	1	.....	.....	+.014	.....	(13)
Fe	5232.940	.957	.....	1	2	fg	+.017	+.014	
	5255.811	.824	.002	4	3	g	+.013	+.036	
	5265.971	.968	.003	6	3	g	−.003	+.021	
Fe	5269.536	.532	.003	5	5	vg	−.004	+.003	(8)
Fe	5270.372	.360	.003	5	5	vg	−.012	.....	(8)
	5283.441	.448	.002	3	3	g	+.007	.....	
Fe	5324.180	.195	.002	2	1	f	+.015	+.015	
Fe	5328.040	.044	.003	4	3	g	+.004	+.004	
Fe	5371.494	.497	.....	1	4	vg	+.003	+.004	
Fe	5397.135	.132	.....	1	3	g	−.003	+.004	
Fe	5405.776	.781	.....	1	3	g	+.005	+.003	
	5409.614	.625	.....	1	2	g	+.011	+.002	
Fe	5429.700	.701	.....	1	3	g	+.001	+.004	
Fe	5446.916	.920	.....	1	3	g	+.004	+.004	
Fe	5455.613	.614	.....	1	1	fp	+.001	+.004	
	5477.697	.732	.....	1	5	f	+.035	.....	
	5512.526	.538	.002	5	7	g	+.012	+.006	(9)
	5514.350	.364	.002	5	7	g	+.014	+.006	(9)
	5514.536	.541	.003	5	5	g	+.005	+.007	(9)
	5565.477	.507	.002	4	3	g	+.030	+.006	
Fe	5615.643	.661	.....	1	3	g	+.018	+.016	
	5644.136	.181	.004	10	4	g	+.045	+.004	
	5662.155	.211	.003	6	3	fg	+.056	+.029	
	5675.413	.474	.003	3	2	fg	+.061	+.031	
	5804.264	.388	.....	1	2	fg	.....	.....	(14)
	5866.451	.508	.002	10	5	g	+.057	+.004	(1)
	5899.292	.338	.002	8	4	g	+.046	+.004	
	5922.109	.160	.001	2	2	fg	+.051	+.006	
	5953.160	.205	.004	10	3	g	+.045	+.003	
	5965.825	.862	.003	10	3	g	+.037	+.005	
	5978.541	.582	.004	10	3	fg	+.041	+.002	
	6085.224	.247	.....	1	4	g	+.023	+.003	
	6091.172	.188	.003	2	3	g	+.016	+.006	
	6126.213	.239	.002	3	3	g	+.026	+.002	
	6258.099	.110	.004	4	5	g	+.011	+.009	(10)
	6258.700	.708	.006	3	6	g	+.008	+.008	(10)
	6261.097	.096	.004	3	5	f	−.001	+.009	(10)

NOTES

Fe values in second column are taken from the values of Burns, Meggers, and Merrill.

The wave-lengths of the Mn lines and of a few Fe lines not given in foregoing reference are taken from Goos.

(1) Special line. Mean of all values.

(2) Meggers' value, 4678.1504.

(3) Close double. These two lines show only one ring system. Measured on long etalon only.

(4) Meggers' value, 4799.9139.

(5) Close double. Only one ring system. Measured on short etalon plates only.

(6) Close double. These two lines show only one ring system. Measured on short etalon plates only.

(7) Meggers' value, 5085.8230.

(8) Close double. Only one ring system. Measured on short etalon plates only.

(9) and (10) These three lines show only one ring system on the long etalon plates. On the short etalon plates they are distinct. Values given here are from short etalon only.

(11) Misprint in Kilby, given as 4522.303, corrected from Kayser's *Handbuch*.

(12) Error in Kilby corrected from Kayser's *Handbuch*.

(13) Two distinct systems of rings on short etalon plate B 90.

(14) Evidently a typographical error in Kilby.

Kilby minus Brown. Column 8 contains the pressure-shift per atmosphere as given by Gale and Adams[1] for Fe and Ti. Column 9 refers to notes at the end of the table.

An examination of column 3 shows that the mean deviation for most of the lines is 0.002 or 0.003 A; dividing this by the square root of the number of plates measured we have an approximate value for the probable error. For the large majority of the lines this is of the order of 0.001 A. From these considerations it is evident that the values in the table compare favorably with other determinations of wave-length by interference methods.[2]

The table includes four pairs of very close lines, each pair showing a single ring system. Three of the pairs showed only on the short etalon plates; one was measured on the long etalon plates only. As it was impossible to decide which one if only one of the two lines was responsible for the ring system, the values as determined for both lines of each pair are given in the table. They are probably interesting rather than valuable. There also appear in the table two sets of triplets in the orange-red region. Here the prism has low dispersion and on the long etalon plates only a single, wide ring system appears for each set of triplets. In both cases,

[1] Gale and Adams, *Astrophysical Journal*, **35**, 10–47, 1912.

[2] Fabry and Buisson, *Journal de Physique*, August, 1913; *Transactions, Solar Union*, **3**, 139–140, 1910; **4**, 46–48, 1913.

however, the short etalon plates show a ring system for each line. The values given in the table are therefore taken from the short etalon plates only.

The differences of wave-length between Kilby's values and mine are for the most part small and these small differences may well be due to pressure-shift. It is assumed that Kilby's arc was operated at atmospheric pressure since he mentions no vacuum. In the orange and red regions, however, the differences are sometimes as much as 0.050 A, but no explanation can be given by the present author as his own values agree among themselves within a few thousandths of an angstrom.

The differences shown in column 7 for the iron lines are some large, some small, but whether large or small most of them agree in sign and magnitude with the pressure-shift given by Gale and Adams and are to be expected from the fact that my arc was worked in vacuum. In so far as they do not agree they point out more clearly the need of further work on the iron arc in vacuo.[1]

The two manganese lines show large differences but here again pressure-shift is probably the cause. Humphreys,[2] grouping all iron lines together and all manganese lines together, finds that on the average the iron lines are shifted 0.025 A by a change of pressure of 12 atmospheres while the manganese lines are shifted 0.033 A by the same change in pressure.

In the case of cadmium, two of the three lines agree well with the latest values of Meggers.[3] The other, λ 4678, differs by 0.005 A. However, the means of the values given by the long and the short etalon differ by this amount and as pointed out in the discussion of the correction for phase the line is known to be complex and is very hard to measure especially on the short etalon plates.

In conclusion the author wishes to express his sincere thanks to Professor Crew and to Professor Fox for their constant interest and many helpful suggestions throughout the course of this work.

NORTHWESTERN UNIVERSITY
April 1922

[1] St. John and Babcock, *Astrophysical Journal*, **53**, 260–299, 1921.

[2] Humphreys, *ibid.*, **6**, 169–232, 1897.

[3] These values are not yet published but were made available through the kindness of the author.

Minor Contributions and Notes

MAJORANA'S THEORY OF GRAVITATION

ABSTRACT

Consequences of Majorana's gravitational results.—The more obvious objections to these results are removed if account is taken of the relativity of the gravitational field and we suppose that the *absorbing effect* is zero for a freely falling body. In fact a confirmation of these results would give a direct demonstration of the *principle of equivalence*. Nevertheless, other very formidable difficulties arise if we attempt to construct a complete theory on this basis. If the results are correct it should be possible to construct a *perpetual-motion machine*.

With reference to Professor Russell's paper on "Majorana's Theory of Gravitation" (*Astrophysical Journal*, **54**, 334), it may not be out of place to consider from a different point of view what theoretical deductions would have to be drawn if the provisional experimental results were confirmed. Russell's results take a different aspect, if account is taken of the relativity of the gravitational field. Clearly a body can only be expected to absorb the gravitational field which is relative to itself, and not the field which is relative to something else. If the body of a man absorbs $1/K$ of the field passing through him, the absorption by "the man in the lift" will be $1/K$ of the enhanced or reduced field which he experiences in the lift, and not $1/K$ of the ordinary G. Now the particles of a planet are all falling freely, so that (except for the minute tide-raising force) the sun's field *relative to them* is zero, and no absorption takes place because there is nothing to absorb; the rear part of the planet is not screened by the front part. In brief, the mercury in Majorana's experiment absorbs because it is not allowed to fall; the planet does not absorb because it is allowed to fall. It is scarcely too much to say that a confirmation of Majorana's result, coupled with the absence of such an effect for the planets as shown by Russell, would give a direct demonstration of the principle of equivalence.

Since the outer parts of the sun and planets are not allowed to fall relatively to the inner parts, the effective mass as measured by

the gravitation-exerting power on surrounding objects would be less than the "true" mass, quite in accordance with Majorana's and Russell's formulae.

While the more obvious objections to Majorana's results are removed by this consideration, it seems that other very formidable difficulties arise if we attempt to construct a complete theory on this basis; so that, although confirming the principle of equivalence, Majorana's absorption would be far from acceptable to gravitational theory as a whole. It may be remarked that since the supposed absorption must be coupled with the condition that the body is not allowed to fall, it may be attributed to the cause which prevents a fall. That is to say, the stresses (which hold up the body) exert a gravitational field, which, superposed on the ordinary attraction of the masses, modify the gradient of the field in a way which is interpreted as absorption. Einstein's theory admits that stresses as well as masses exert gravitation, and a very minute absorption by a supported body would, I think, be consistent with his theory; but his results, based on the conservation of energy and momentum, are of a far smaller order of magnitude than Majorana's. This is not surprising since, if Majorana's theory is correct, it should be easy to construct a perpetual-motion machine.

A. S. EDDINGTON